Secondary

MATHEMATICS

Class-VII

Publication Division

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Contents

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CHAPTER 1 RATIONAL NUMBERS

INTRODUCTION

Do you remember number systems?

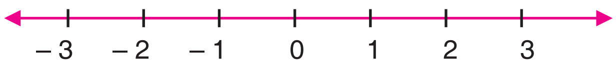
(i) Numbers 1, 2, 3, 4, . . . which we use for counting, form the **system of natural numbers**.

Natural numbers are
1, 2, 3, 4 ....

(ii) Natural numbers along with zero, form the **system of whole numbers**.

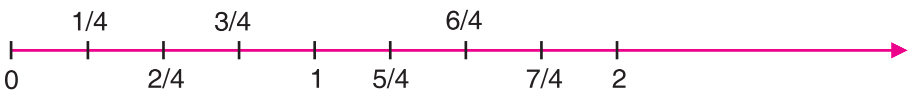
Whole numbers are
0, 1, 2, 3, 4, 5..........

(iii) Collection of natural numbers, their opposites along with zero is called the **system of integers**.



Integers are
......., – 3, – 2, – 1, 0, 1, 2, 3 ....

(iv) A part of a whole is a fraction. **Fraction** is the ratio of two natural numbers, e.g. $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{5}{4}, \frac{6}{4}, \ldots$



Properties of fractions

(a) If $\frac{p}{q}$ is a fraction, then for any natural number m,

$$\boxed{\frac{p}{q} = \frac{p \times m}{q \times m}}$$

(b) If $\frac{p}{q}$ is a fraction and a natural number m is a common divisor of p and q, then

$$\boxed{\frac{p}{q} = \frac{p \div m}{q \div m}}$$

(c) Two fractions $\frac{p}{q}$ and $\frac{r}{s}$ are said to be equivalent if

$$\boxed{p \times s = q \times r}$$

(d) A fraction $\frac{p}{q}$ is said to be in its simplest or lowest form if

p and q have no common factor other than 1.

(e) Fractions can be compared as:

(i) $\frac{p}{q} < \frac{r}{s}$ if $p \times s < q \times r$

(ii) $\frac{p}{q} = \frac{r}{s}$ if $p \times s = q \times r$

(iii) $\frac{p}{q} > \frac{r}{s}$ if $p \times s > q \times r$

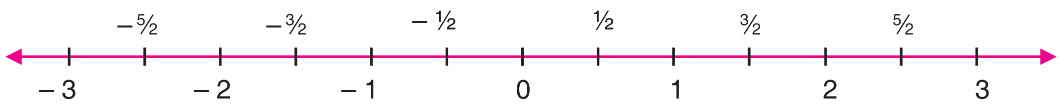
Let us do some problems to revise our memory.

Simplify the following:

1. (– 212) + 384 – (– 137)
2. (– 9) × [7 + (– 11)]
3. (– 12) × (– 10) × 6 × (– 1)
4. (– 108) ÷ (– 12)
5. (– 1331) ÷ 11
6. – 72 (– 15 – 37 – 18)

RATIONAL NUMBERS

In Class–VI, we have dealt with negative integers. In the same way, we shall be introducing negative fractions, e.g. corresponding to 1/2 we have negative fraction – 1/2.



Fractions with corresponding negative fractions and zero constitute the **system of rational numbers.**

The word ‘rational’ comes from the word ‘ratio’.

Any number which can be expressed in the form of p/q, where p and q are integers and q ≠ 0 is known as a **Rational Number**.

See the following rational numbers.

$$\frac{1}{5} \qquad \frac{5}{-2} \qquad \frac{2}{3} \qquad \frac{-1}{5} \qquad \frac{-2}{3} \qquad \frac{-2}{-3} \qquad \frac{1}{-4}$$

Positive Rational Numbers

$$\frac{1}{5}, \quad \frac{2}{3}, \quad \frac{-2}{-3}$$

The rational numbers are said to be **positive** if signs of numerator and denominator are the same.

Negative Rational Numbers

$$\frac{5}{-2}, \quad \frac{-1}{5}, \quad \frac{-2}{3}, \quad \frac{1}{-4}$$

The rational numbers are said to be **negative** if signs of numerator and denominator are not the same.

Remember

- Every fraction is a rational number, but every rational number need not be a fraction, e.g. $\frac{-4}{7}, \frac{0}{3}$ are not fractions as fractions are part of a whole which are always positive.
- All the integers are rational numbers. Integers – 50, 15, 0 can be written as, $\frac{-50}{1}, \frac{15}{1}, \frac{0}{1}$ respectively.

Worksheet 1

1. **Which of the following are rational numbers?**

   (i) -3 (ii) $-\frac{2}{3}$ (iii) $\frac{4}{0}$ (iv) $\frac{0}{-5}$

2. **Write down the rational numbers in the form $\frac{p}{q}$ whose numerators and denominators are given below:**

   (i) $(-5) \times 4$ and $-5 + 4$ (ii) $64 \div 4$ and $32 - 18$

3. **Which of the following are positive rational numbers?**

(i) $\frac{-2}{9}$ (ii) $\frac{3}{-5}$ (iii) $\frac{4}{9}$ (iv) $\frac{-3}{-19}$ (v) $\frac{0}{-3}$

4. **Answer the following:**

(i) Which integer is neither positive nor negative?

(ii) A rational number can always be written as $\frac{p}{q}$. Is it necessary that any number written as $\frac{q}{p}$ is a rational number?

5. **State whether the following statements are true. If not, justify your answer with an example.**

(i) Every whole number is a natural number.
(ii) Every natural number is an integer.
(iii) Every integer is a whole number.
(iv) Every integer is a rational number.
(v) Every rational number is a fraction.
(vi) Every fraction is a rational number.

PROPERTIES OF RATIONAL NUMBERS

Property I. Two rational numbers $\frac{p}{q}$ and $\frac{r}{s}$ are said to be **equivalent** if

$$p \times s = r \times q.$$

To explain the property, let us take few examples.

Example 1: Show that $\frac{4}{-7}$ and $\frac{8}{-14}$ are equivalent rational numbers.

Solution: $4 \times (-14) = -56 = 8 \times (-7)$.

Hence, $\frac{4}{-7}$ and $\frac{8}{-14}$ are equivalent rational numbers.

Example 2: Show that $\frac{5}{8}$ and $\frac{-15}{24}$ are not equivalent rational numbers.

Solution: $5 \times 24 = 120$ and $8 \times (-15) = -120$.

Hence, $5 \times 24 \neq 8 \times (-15)$.

Therefore, the given rational numbers are not equivalent.

Property II. If $\frac{p}{q}$ is a rational number and m be any integer different from zero, then

$$\frac{p}{q} = \frac{p \times m}{q \times m}.$$

Example 3: Write three rational numbers which are equivalent to $\frac{3}{5}$.

Solution: To find equivalent rational numbers, multiply numerator and denominator by any same non-zero integer.

$$\frac{3 \times 2}{5 \times 2} = \frac{6}{10} \quad \text{(Multiply numerator and denominator by 2)}$$

$$\frac{3 \times (-3)}{5 \times (-3)} = \frac{-9}{-15} \quad \text{(Multiply numerator and denominator by – 3)}$$

$$\frac{3 \times 5}{5 \times 5} = \frac{15}{25} \quad \text{(Multiply numerator and denominator by 5)}$$

Hence, $\frac{6}{10}, \frac{-9}{-15}$ and $\frac{15}{25}$ are three rational numbers equivalent to $\frac{3}{5}$.

Example 4: Express $\frac{-4}{7}$ as a rational number with (i) numerator 12 (ii) denominator 28.

Solution: (i) To get numerator 12, we must multiply – 4 by – 3.

Hence, $$\frac{(-4) \times (-3)}{7 \times (-3)} = \frac{12}{-21}$$

Therefore, the required rational number is $\frac{12}{-21}$.

(ii) To get denominator 28, we must multiply the given denominator 7 by 4.

i.e. $$\frac{(-4) \times 4}{7 \times 4} = \frac{-16}{28}$$

Hence, the required rational number is $\frac{-16}{28}$.

Property III. If $\frac{p}{q}$ is a rational number and m is a common divisor of p and q then

$$\frac{p}{q} = \frac{p \div m}{q \div m}$$

Example 5: Express $\frac{-21}{49}$ as a rational number with denominator 7.

Solution: To get denominator 7, we must divide 49 by 7.

Therefore, $\frac{-21 \div 7}{49 \div 7} = \frac{-3}{7}$.

Hence, $\frac{-3}{7}$ is the required rational number.

Worksheet 2

1. In each of the following cases, show that the rational numbers are equivalent.

(i) $\frac{4}{9}$ and $\frac{44}{99}$ (ii) $\frac{7}{-3}$ and $\frac{35}{-15}$ (iii) $\frac{-3}{5}$ and $\frac{-12}{20}$

2. In each of the following cases, show that rational numbers are not equivalent.

(i) $\frac{4}{9}$ and $\frac{16}{27}$ (ii) $\frac{-100}{3}$ and $\frac{300}{9}$ (iii) $\frac{3}{-17}$ and $\frac{8}{-51}$

3. Write three rational numbers, equivalent to each of the following:

(i) $\frac{4}{7}$ (ii) $\frac{36}{108}$ (iii) $\frac{-5}{-7}$ (iv) $\frac{-72}{180}$

4. Express $\frac{3}{5}$ as rational number with numerator,

(i) -21 (ii) 150

5. Express $\frac{4}{-7}$ as a rational number with denominator,

(i) 84 (ii) -28

6. Express $\frac{90}{216}$ as a rational number with numerator 5.

7. Express $\frac{-64}{256}$ as a rational number with denominator 8.

8. Find equivalent forms of the rational numbers having a common denominator in each of the following collections of rational numbers.

(i) $\frac{2}{5}, \frac{6}{13}$ (ii) $\frac{1}{7}, \frac{2}{8}, \frac{3}{14}$ (iii) $\frac{5}{12}, \frac{7}{4}, \frac{9}{60}, \frac{11}{3}$

STANDARD FORM OF A RATIONAL NUMBER

Let us try to express a rational number in the simplest form with positive denominator.

Example 6: Express $\frac{16}{-24}$ in the simplest form with its denominator as positive.

Solution: **Step 1.** Convert denominator into positive by multiplying numerator and denominator by –1.

$$\frac{(16)\times(-1)}{(-24)\times(-1)} = \frac{-16}{24}$$

Step 2. Find HCF of 16 and 24, which is 8 in this case, and divide numerator and denominator by it.

$$\frac{-16 \div 8}{24 \div 8} = \frac{-2}{3}$$

The example given above explains that every rational number $\frac{p}{q}$ can be put in the simplest form with positive denominator. This form of the rational number is called its **standard form**. For this, we take the following steps.

Step 1. Make the denominator positive.

Step 2. Find the HCF m of p and q. If m = 1, then $\frac{p}{q}$ is the required form.

Step 3. If m ≠ 1, then divide both the numerator and the denominator by m. The rational number $\frac{p \div m}{q \div m}$ so obtained is the required standard form.

Note:

The numbers $\frac{-p}{q}$ and $\frac{p}{-q}$ represent the same rational number.

A rational number $\frac{p}{q}$ is said to be in the standard form if q is positive and the integers 'p' and 'q' have their highest common factor as 1.

Example 7: Express $\frac{-22}{-55}$ in the standard form.

Solution: **Step 1.** $\frac{-22\times(-1)}{-55\times(-1)} = \frac{22}{55}$

Step 2. HCF of 22 and 55 is 11.

$\frac{22\div 11}{55\div 11} = \frac{2}{5}$ which is the standard form.

Example 8: Find x such that the rational numbers in each of the following pairs are equivalent.

(i) $\frac{x}{12}, \frac{5}{6}$ (ii) $\frac{15}{x}, \frac{-3}{8}$

Solution: (i) $\frac{x}{12}, \frac{5}{6}$ will be equivalent if

$$6 \times x = 5 \times 12$$

$$x = \frac{5\times \cancel{12}^{2}}{\cancel{6}} = 5 \times 2 = 10$$

Hence, $x = 10$.

(ii) $\frac{15}{x}, \frac{-3}{8}$ will be equivalent if

$$15 \times 8 = (-3) \times x$$

$$x = \frac{15\times 8}{-3} = -5 \times 8 = -40$$

Hence, $x = -40$.

Example 9: Fill in the blanks: $\frac{-3}{5} = \frac{6}{\_\_\_} = \frac{\_\_\_}{-15}$

Solution: In the first two given rational numbers, we have to find the number which when multiplied by – 3 gives the product 6. Here, the number shall be 6 ÷ (– 3) = – 2. Now, we multiply both numerator and denominator of the given rational number by – 2.

We get $\frac{-3}{5} = \frac{(-3)\times(-2)}{5\times(-2)} = \frac{6}{-10}$

To get denominator – 15,

$$\frac{-3}{5} = \frac{(-3)\times(-3)}{5\times(-3)} = \frac{9}{-15}$$ (Multiply numerator and denominator by – 3)

Thus, $\frac{-3}{5} = \frac{6}{-10} = \frac{9}{-15}$

Worksheet 3

1. Write the following rational numbers in standard form.

(i) $\frac{33}{77}$ (ii) $\frac{64}{-20}$ (iii) $\frac{-27}{-15}$ (iv) $\frac{-105}{98}$

2. Find x such that the rational numbers in each of the following pairs, become equivalent.

(i) $\frac{9}{-5}, \frac{x}{10}$ (ii) $\frac{8}{7}, \frac{x}{-35}$ (iii) $\frac{36}{x}, 2$ (iv) $\frac{x}{6}, -13$

3. Check whether the following rational numbers are in standard form. If not, write them in standard form.

(i) $\frac{-3}{19}$ (ii) $\frac{4}{-7}$ (iii) $\frac{14}{35}$ (iv) $\frac{8}{-72}$

4. Fill in the blanks.

(i) $\frac{2}{7} = \frac{8}{\_\_\_} = \frac{\_\_\_}{-63}$ (ii) $\frac{36}{\_\_\_} = \frac{-4}{9} = \frac{-84}{\_\_\_}$ (iii) $\frac{105}{\_\_\_} = \frac{\_\_\_}{-99} = \frac{-5}{-11}$

ABSOLUTE VALUE OF A RATIONAL NUMBER

We have studied in Class–VI that absolute value of an integer is its numerical value without taking the sign into account, e.g. $|-3| = 3, |3| = 3, |0| = 0$

The absolute value of a rational number is written in the following ways.

Absolute value of $\frac{4}{5}$ is $\left|\frac{4}{5}\right| = \frac{4}{5}$

Absolute value of $\frac{-4}{5}$ is $\left|\frac{-4}{5}\right| = \frac{4}{5}$

Absolute value of $\frac{4}{-5}$ is $\left|\frac{4}{-5}\right| = \frac{4}{5}$

Absolute value of $\frac{-4}{-5}$ is $\left|\frac{-4}{-5}\right| = \frac{4}{5}$

Remember

- Absolute value of every rational number other than zero is positive.
- The absolute value of zero is zero itself.
- Absolute value of a rational number is greater than or equal to the number itself.

Worksheet 4

1. Find the absolute value of the following rational numbers.

(i) $\frac{1}{-5}$ (ii) $\frac{7}{9}$ (iii) $\frac{0}{-4}$ (iv) $\frac{-3}{-2}$

2. Compare the absolute values of the rational numbers in the following pairs.

(i) $\frac{3}{-7}, \frac{-3}{7}$ (ii) $\frac{-5}{7}, \frac{4}{3}$ (iii) $\frac{-4}{5}, -3$

3. Find all the rational numbers whose absolute value is–

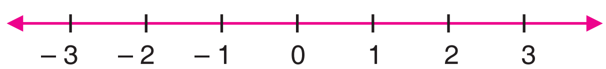
(i) $\frac{2}{5}$ (ii) 0 (iii) $\frac{3}{4}$

REPRESENTATION OF RATIONAL NUMBERS ON A NUMBER LINE

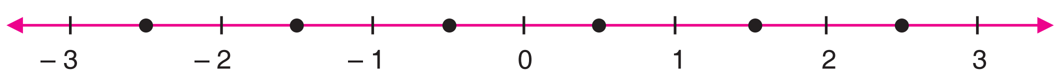
You have dealt with the definition and properties of rational numbers. Now, you will learn how to plot rational numbers on a number line.

Let us mark the rational numbers $\frac{-5}{2}, \frac{-3}{2}, \frac{-1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ on the number line.

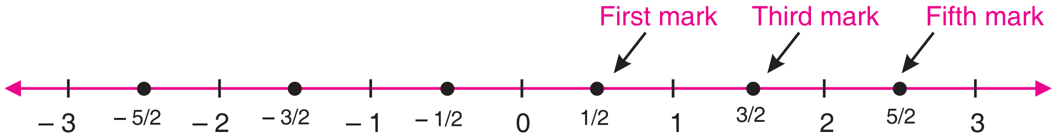
Step 1. Mark integers on the number line.



Step 2. Divide each unit segment into two equal parts (equal to denominator).



Step 3. $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ are represented by first, third and fifth mark respectively lying to the right of zero.



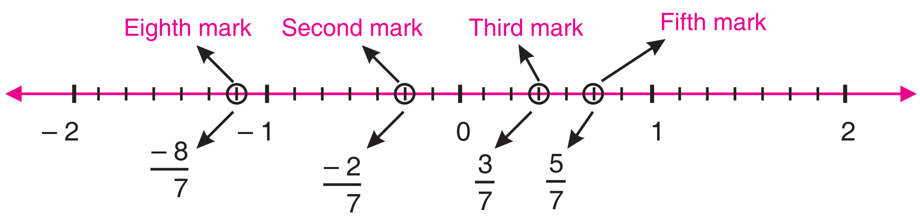
$\frac{-1}{2}, \frac{-3}{2}, \frac{-5}{2}$ are represented by first, third and fifth mark respectively lying to the left of zero.

Example 10: Represent $\frac{3}{7}, \frac{5}{7}, \frac{-8}{7}, \frac{-2}{7}$ on number line.

Solution: **Step 1.** Mark integers on number line.

Step 2. Divide each unit segment into seven equal parts.

Step 3. Third and fifth mark on right side of zero represent $\frac{3}{7}$ and $\frac{5}{7}$. Eighth and second mark on left side of zero represent $\frac{-8}{7}$ and $\frac{-2}{7}$ respectively.



Worksheet 5

1. State whether the following statements are true. If not, justify your answer.

(i) On a number line, all the numbers to the right of zero are positive.

(ii) Rational number $\frac{-7}{-19}$ lies to the left of zero on the number line.

(iii) On a number line, numbers become progressively larger as we move away from zero.

(iv) Rational numbers $\frac{2}{3}$ and $\frac{-2}{3}$ are at equal distance from zero.

(v) On a number line, number lying left to a given number is greater.

2. Mark the following rational numbers on number line.

(i) $\frac{4}{5}$ (ii) $\frac{-8}{3}$ (iii) $\frac{5}{2}$ (iv) $\frac{-7}{3}$

3. Represent the following rational numbers on a number line.

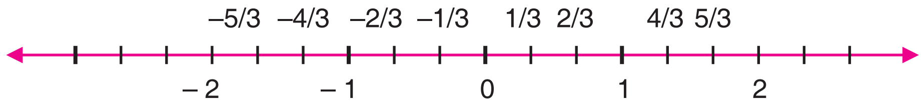
(i) $\frac{-3}{5}$ (ii) $\frac{2}{-3}$ (iii) $\frac{3}{4}$ (iv) $\frac{-4}{-7}$

COMPARING RATIONAL NUMBERS

Rational numbers can be compared in two different ways.

I. BY REPRESENTING ON NUMBER LINE

Rational numbers can be compared easily when they are represented on the number line.



Any number on the number line is greater than any other number lying to the left of it.

Any number on a number line is less than any other number lying to the right of it.

Therefore, from the above number line it is clear that

$$\frac{2}{3} < \frac{5}{3}, \quad 1 < \frac{4}{3}, \quad \frac{-5}{3} < \frac{-1}{3}, \quad \frac{-4}{3} < \frac{1}{3}, \text{ etc.}$$

and $$\frac{4}{3} > \frac{1}{3}, \frac{4}{3} > \frac{-2}{3}, \quad \frac{-1}{3} > \frac{-4}{3}, \text{ etc.}$$

II. WITHOUT REPRESENTING ON NUMBER LINE

Without representing the rational numbers on the number line, we can compare them by the method similar to the one used for fractional numbers.

If two rational numbers have the same positive denominator, the number with the larger numerator will be greater than the one with smaller numerator.

Example 11: Compare,

(i) $\frac{2}{7}$ and $\frac{5}{7}$ (ii) $\frac{-6}{17}$ and $\frac{-13}{17}$

Solution: (i) The rational numbers $\frac{2}{7}$ and $\frac{5}{7}$ have same denominator, therefore, smaller the numerator, smaller will be the rational number. Since 2 < 5, therefore, $\frac{2}{7} < \frac{5}{7}$.

(ii) $\frac{-6}{17}$ and $\frac{-13}{17}$ have the same denominator. Therefore, we shall compare the numerators–

$$-6 > -13$$

Therefore, $\frac{-6}{17} > \frac{-13}{17}$.

> **If two rational numbers have different denominators, then first make denominators equal and then compare.**

Example 12: Compare $\frac{7}{5}$ and $\frac{8}{7}$.

Solution: First, convert the rational numbers to have the same positive denominator.

$$\left.\begin{aligned} \frac{7}{5} &= \frac{7\times 7}{5\times 7} = \frac{49}{35} \\ \frac{8}{7} &= \frac{8\times 5}{7\times 5} = \frac{40}{35} \end{aligned}\right] \quad \text{(Denominators are same)}$$

Now, compare $\frac{49}{35}$ and $\frac{40}{35}$

As $49 > 40$, therefore, $\frac{49}{35} > \frac{40}{35}$

Hence, $\frac{7}{5} > \frac{8}{7}$.

Example 13: Compare the rational numbers $\frac{-4}{9}$ and $\frac{5}{-6}$.

Solution: First write $\frac{5}{-6}$ in standard form, i.e. $\frac{-5}{6}$.

Now, convert them to have the same denominator.

$$\frac{-4}{9} \times \frac{2}{2} = \frac{-8}{18}$$

$$\frac{-5}{6} \times \frac{3}{3} = \frac{-15}{18}$$

Now, compare $\frac{-8}{18}$ and $\frac{-15}{18}$

Since, numerator $-8 > -15$, therefore, $\frac{-8}{18} > \frac{-15}{18}$.

Hence, $\frac{-4}{9} > \frac{5}{-6}$.

There is yet another method to compare two rational numbers $\frac{p}{q}$ and $\frac{r}{s}$ with unequal denominators. It is assumed that q and s are both positive integers.

To compare $\frac{p}{q}$ and $\frac{r}{s}$, we may compare ps and qr $\left(\frac{p}{q} \times \frac{r}{s}\right)$

Find products ps and qr.

If ps > qr then $\frac{p}{q} > \frac{r}{s}$.

If ps < qr then $\frac{p}{q} < \frac{r}{s}$.

Example 14: Compare $\frac{5}{3}$ and $\frac{2}{7}$.

Solution: $\frac{5}{3} \times \frac{2}{7}$

The products are $5 \times 7 = 35$ and $3 \times 2 = 6$

Since, $35 > 6$, therefore, $\frac{5}{3} > \frac{2}{7}$.

Example 15: Compare $\frac{-5}{7}$ and $\frac{4}{-9}$.

Solution: First write $\frac{4}{-9}$ in standard form as $\frac{-4}{9}$.

Now, find the products $\frac{-5}{7} \times \frac{-4}{9}$

The products are $-5 \times 9 = -45$ and $7 \times (-4) = -28$

Since, $-45 < -28$, therefore, $\frac{-5}{7} < \frac{-4}{9}$.

Worksheet 6

1. Determine which rational number is greater in each case.

(i) $\frac{5}{8}, \frac{-3}{7}$

(ii) $\frac{2}{3}, \frac{8}{9}$

(iii) $\frac{-4}{3}, \frac{-6}{7}$

(iv) $\frac{-8}{3}, \frac{19}{-6}$

(v) $\frac{-3}{-13}, \frac{-5}{-21}$

(vi) $\frac{-7}{11}, \frac{5}{-8}$

2. Find the value of x, if–

(i) $\frac{3}{-5} = \frac{x}{15}$

(ii) $\frac{9}{15} = \frac{x}{-50}$

(iii) $\frac{36}{x} = -4$

(iv) $\frac{7}{-x} = 7$

3. Compare the rational numbers.

(i) $\frac{-2}{9}, \frac{8}{-36}$

(ii) $\frac{5}{9}, \frac{4}{6}$

(iii) $\frac{-7}{-8}, \frac{14}{17}$

(iv) $\frac{-4}{7}, \frac{5}{-9}$

(v) $\frac{-5}{8}, \frac{-3}{4}$

(vi) $\frac{6}{7}, \frac{-54}{-63}$

4. Arrange the following in ascending order.

(i) $\frac{4}{7}, \frac{5}{9}, \frac{2}{5}$

(ii) $\frac{-3}{4}, \frac{-5}{-12}, \frac{-7}{16}$

5. Arrange the following in descending order.

(i) $\frac{2}{5}, \frac{-1}{2}, \frac{8}{-15}, \frac{-3}{-10}$

(ii) $\frac{-7}{10}, \frac{8}{-15}, \frac{19}{30}, \frac{-2}{-5}$

VALUE BASED QUESTIONS

1. **Sukhdev, a farmer, had a son and a daughter. He decided to divide his property among his children. He gave $\frac{2}{5}$ of the property to his son and $\frac{4}{10}$ to his daughter, and rest to a charitable trust.**

   (a) Whose share was more, son's or daughter's?

   (b) What do you feel about Sukhdev's decision? Which values are exhibited here?

2. **Kavita along with her family was planning a vacation at a hill station. But, they were confused where to go. Kavita's mother asked her to find out the maximum temperature of few hill stations for deciding on the place to visit. She checked the weather report on the internet and found that—**

   Simla's temperature = $\left(\frac{-7}{2}\right)^{\circ}$ C

   Dalhousie's temperature = – 5°C

   Manali's temperature = $\left(\frac{-8}{5}\right)^{\circ}$ C

   (a) Arrange the temperatures of these hill stations in ascending order.

   (b) Which place will they decide to visit?

   (c) What value is exhibited in the above situation?

BRAIN TEASERS

1. **A. Tick (✓) the correct option.**

   (a) The value of x such that $\frac{-3}{8}$ and $\frac{x}{-24}$ are equivalent rational numbers is–

   (i) 64 (ii) – 64

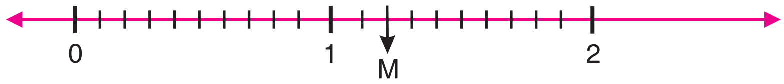
   (iii) – 9 (iv) 9

   (b) Which of the following is a negative rational number?

   (i) $\frac{-15}{-4}$ (ii) 0

   (iii) $\frac{-5}{7}$ (iv) $\frac{4}{9}$

(c) In the given number line, which of the following rational numbers does the point M represent?



(i) $\frac{2}{8}$ (ii) $\frac{6}{5}$ (iii) $\frac{2}{3}$ (iv) $\frac{12}{5}$

(d) Which is the greatest rational number out of $\frac{5}{-11}, \frac{-5}{12}, \frac{5}{-17}$?

(i) $\frac{5}{-11}$ (ii) $\frac{-5}{12}$

(iii) $\frac{5}{-17}$ (iv) cannot be compared

(e) Which of the following rational numbers is the smallest?

(i) $\left|\frac{7}{11}\right|$ (ii) $\left|\frac{-8}{11}\right|$ (iii) $\left|\frac{-2}{11}\right|$ (iv) $\left|\frac{-9}{-11}\right|$

B. Answer the following questions.

(a) Find the average of the rational numbers $\frac{4}{5}, \frac{2}{3}, \frac{5}{6}$.

(b) How will you write $\frac{12}{-18}$ in the standard form?

(c) How many rational numbers are there between any two rational numbers?

(d) On the number line, the rational number $\frac{-5}{-7}$ lies on which side of zero?

(e) Express $\frac{-7}{-8}$ as a rational number with denominator 40.

2. State whether the following statements are true. If not, then give an example in support of your answer.

(i) If $\frac{p}{q} > \frac{r}{s}$ then $\left|\frac{p}{q}\right| > \left|\frac{r}{s}\right|$

(ii) If $|x| = |y|$ then $x = y$

(iii) $\frac{p}{q}$ is a non-zero rational number in standard form. It is necessary that rational number $\frac{q}{p}$ will also be in standard form.

3. **Represent $5\frac{1}{3}$ and $\frac{-29}{4}$ on a number line.**

4. **Arrange the following rational numbers in descending order.**

   $\frac{-3}{10}, \frac{-7}{-5}, \frac{9}{-15}, \frac{18}{30}$

5. **On a number line, what is the length of the line-segment joining,**

   (i) 3 and – 3? (ii) $\frac{1}{2}$ and $\frac{-1}{2}$?

   (iii) $\frac{1}{2}$ and $2\frac{1}{2}$? (iv) $\frac{-1}{2}$ and $-2\frac{1}{2}$?

6. **Find the values of x in each of the following:**

   (i) $\frac{23}{x} = \frac{2}{-8}$ (ii) $\frac{x}{9} = \frac{19}{3}$ (iii) $\frac{15}{-x} = \frac{1}{-7}$

7. **Compare the numbers in each of following pairs of numbers.**

   (i) $\frac{-5}{7}, \frac{9}{-13}$ (ii) $\frac{-4}{9}, \frac{-3}{7}$ (iii) $\frac{-3}{-5}, \frac{12}{20}$

   (iv) $\left|\frac{-4}{5}\right|, \left|\frac{-5}{4}\right|$ (v) $\left|\frac{5}{7}\right|, \left|\frac{-15}{21}\right|$ (vi) $\left|\frac{-8}{-9}\right|, \left|\frac{-3}{9}\right|$

8. **Fill in the following blank squares.**

   (i) $\frac{3}{5} = \frac{138}{\square}$ (ii) $\frac{7}{9} = \frac{\square}{108}$

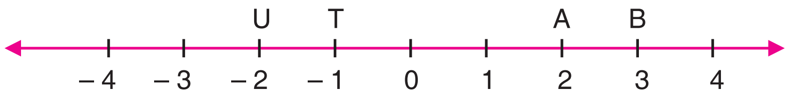
   (iii) $\frac{\square}{-15} = \frac{48}{90}$ (iv) $\frac{121}{\square} = \frac{-11}{12}$

HOTS

The points P, Q, R, S, T, U, A and B are on the number line representing integers such that—

TR = RS = SU and AP = PQ = QB

Locate and write the rational numbers represented by points P, Q, R, and S.



YOU MUST KNOW

1. A number of the form $\frac{p}{q}$ is called a fraction, if p and q are natural numbers. If p and q are integers and $q \neq 0$, then it is said to be a rational number.
2. Every integer and fraction is a rational number but the converse may not be true.
3. A rational number is said to be positive if both numerator and denominator are of same sign. If numerator and denominator are of opposite signs, then rational number is said to be negative.
4. If $\frac{p}{q}$ be a rational number and m be any integer different from zero, then $\frac{p}{q} = \frac{p \times m}{q \times m}$.
5. If $\frac{p}{q}$ be a rational number and m be a common divisor of p and q, then $\frac{p}{q} = \frac{p \div m}{q \div m}$.
6. A rational number $\frac{p}{q}$ is said to be in standard form if q is positive and HCF of p and q is 1.
7. Two rational numbers $\frac{p}{q}$ and $\frac{r}{s}$ are said to be equivalent (equal) if $p \times s = q \times r$.
8. Every rational number can be represented on the number line.
9. If $\frac{p}{q}$ and $\frac{r}{s}$ are two rational numbers with q and s positive integers then $\frac{p}{q} > \frac{r}{s}$ if $p \times s > q \times r$, $\frac{p}{q} < \frac{r}{s}$ if $p \times s < q \times r$ and $\frac{p}{q} = \frac{r}{s} =$ if $p \times s = q \times r$.
10. Every rational number has an absolute value which is greater than or equal to zero.

CHAPTER 2

OPERATIONS ON RATIONAL NUMBERS

In Class-V and VI, we have dealt with the operations (addition, subtraction, multiplication, division) on fractions and integers. Now, we shall study these operations and their properties in case of Rational Numbers.

ADDITION OF RATIONAL NUMBERS

I. When the rational numbers have the same denominator.

Example 1: Add $\frac{3}{7}$ and $\frac{6}{7}$.

Solution: $\frac{3}{7}+\frac{6}{7} = \frac{3+6}{7}$ ← Addition of numerators, ← Common denominator

$= \frac{9}{7}$

Add numerators of the rational numbers and then divide by the common denominator.

Example 2: Find the sum of (i) $\frac{3}{10}, \frac{-7}{10}$ (ii) $\frac{9}{-11}, \frac{8}{-11}$

Solution: (i) $\frac{3}{10}+\frac{-7}{10}=\frac{3+(-7)}{10}=\frac{3-7}{10}=\frac{-4}{10}=\frac{-2}{5}$ (standard form)

(ii) $\frac{9}{-11}+\frac{8}{-11}=\frac{9+8}{-11}=\frac{17}{-11}=\frac{-17}{11}$ (standard form)

II. When the rational numbers have different denominators.

Example 3: Add $\frac{1}{4}$ and $\frac{2}{3}$.

Solution: $\frac{1}{4}+\frac{2}{3}$

LCM of denominators 4 and 3 is 12.

$$\frac{1}{4} = \frac{1 \times 3}{4 \times 3} = \frac{3}{12}$$

$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

denominators are same.

Add $\frac{3}{12} + \frac{8}{12} = \frac{3+8}{12} = \frac{11}{12}$

Take the LCM of denominators and then add the rational numbers.

Example 4: Find the sum of (i) $\frac{-3}{11}, \frac{-2}{7}$ (ii) $\frac{7}{-16}, \frac{-3}{4}$

Solution: (i) $\frac{-3}{11} + \frac{(-2)}{7}$

LCM of denominators 11 and 7 is 77.

$$\frac{-3}{11} = \frac{-3 \times 7}{11 \times 7} = \frac{-21}{77}$$

$$\frac{-2}{7} = \frac{-2 \times 11}{7 \times 11} = \frac{-22}{77}$$

Now add $\frac{-21}{77} + \frac{-22}{77} = \frac{(-21) + (-22)}{77}$

$$= \frac{-21 - 22}{77} = \frac{-43}{77}.$$

(ii) $\frac{7}{(-16)} + \frac{(-3)}{4}$

$$\frac{7}{-16} = \frac{7 \times (-1)}{-16 \times (-1)} = \frac{-7}{16}$$ (standard form)

Now, we have to add, $\frac{-7}{16}$ and $\frac{-3}{4}$.

LCM of denominators 16 and 4 is 16.

$$\frac{-7}{16} + \frac{-3}{4} = \frac{-7}{16} + \frac{-3 \times 4}{4 \times 4}$$

$$= \frac{-7}{16} + \frac{(-12)}{16}$$

$$= \frac{-7-12}{16} = \frac{-19}{16}.$$

Note:

From above examples, it is clear that the sum of any two rational numbers is also a rational number.

PROPERTIES OF ADDITION OF RATIONAL NUMBERS

Like other numbers (natural numbers, whole numbers, fractions and integers), addition of rational numbers also satisfy the following properties.

1. **Verify that the sum of two rational numbers, $\frac{-2}{7}$ and $\frac{1}{4}$ remains the same even if the order of addends is changed.**

First, find the sum $\frac{-2}{7}+\frac{1}{4}$

$$\frac{-2\times4}{7\times4}+\frac{1\times7}{4\times7} \quad \text{(LCM of 7 and 4 is 28)}$$

$$\frac{-8}{28}+\frac{7}{28}=\frac{-8+7}{28}=\frac{-1}{28}$$

Now, change the order and find the sum.

$$\frac{1}{4}+\frac{-2}{7} = \frac{1\times7}{4\times7}+\frac{-2\times4}{7\times4}=\frac{7}{28}+\frac{(-8)}{28}=\frac{7+(-8)}{28}=\frac{-1}{28}$$

What do you observe?

Property 1: **The sum remains the same even if we change the order of addends, i.e. for two rational numbers x and y, x + y = y + x.**
This is commutative law of addition.

2. **Verify that the sum of three rational numbers $\frac{1}{2}, \frac{2}{3}$ and $\frac{5}{4}$ remains the same even after changing the grouping.**

$$\left(\frac{1}{2}+\frac{2}{3}\right)+\frac{5}{4} \leftarrow \left(\text{First add } \frac{1}{2} \text{ and } \frac{2}{3}\right)$$

$$= \left(\frac{1\times3}{2\times3}+\frac{2\times2}{3\times2}\right)+\frac{5}{4}$$

$$\frac{1}{2}+\left(\frac{2}{3}+\frac{5}{4}\right) \leftarrow \text{(Grouping is changed)}$$

$$= \frac{1}{2}+\left(\frac{2\times4}{3\times4}+\frac{5\times3}{4\times3}\right)$$

$= \left(\frac{3}{6}+\frac{4}{6}\right)+\frac{5}{4}$

$= \left(\frac{3+4}{6}\right)+\frac{5}{4}$

$= \frac{7}{6}+\frac{5}{4}$

$= \frac{7\times 2}{6\times 2}+\frac{5\times 3}{4\times 3}$

$= \frac{14}{12}+\frac{15}{12}$

$= \frac{14+15}{12}$

$= \frac{29}{12}$

$= \frac{1}{2}+\left(\frac{8}{12}+\frac{15}{12}\right)$

$= \frac{1}{2}+\left(\frac{8+15}{12}\right)$

$= \frac{1}{2}+\frac{23}{12}$

$= \frac{1\times 6}{2\times 6}+\frac{23}{12}$

$= \frac{6}{12}+\frac{23}{12}$

$= \frac{6+23}{12}$

$= \frac{29}{12}$

Same

Property 2: **Sum of three rational numbers remains same even after changing the grouping of the addends, i.e. if x, y and z are three rational numbers, then $(x + y) + z = x + (y + z)$**
This is known as associative law of addition.

3. Find the sum of $\frac{5}{3}$ and 0.

$$\frac{5}{3}+0=\frac{5}{3}+\frac{0}{3}=\frac{5+0}{3}=\frac{5}{3}$$

Similarly, $0+\frac{(-27)}{23}=\frac{0}{23}+\frac{(-27)}{23}=\frac{0-27}{23}=\frac{-27}{23}$

Property 3: **When zero is added to any rational number, the sum is the rational number itself, i.e. if x is a rational number, then $0 + x = x + 0 = x$.**
Zero is called the identity element of addition.

4. Find the sum of $\frac{7}{9}$ and $\frac{-7}{9}$.

$$\frac{7}{9}+\frac{-7}{9}=\frac{7+(-7)}{9}=\frac{0}{9}=0$$

Zero is the identity element of addition.

Similarly, $\frac{-2}{3}+\frac{2}{3}=\frac{(-2)+2}{3}=\frac{0}{3}=0$

Property 4: **Every rational number has an additive inverse such that their sum is equal to zero. If x is a rational number, then – x is a rational number such that x + (– x) = 0.**
– x is called additive inverse of x. It is also called the negative of x.

Following are a few examples to illustrate these properties.

Example 5: Simplify, $\frac{3}{7}+\frac{5}{7}+\frac{-3}{7}+\frac{-5}{7}$.

Solution: To find the sum of three or more rational numbers, we may arrange them in any order we like. The arrangement does not alter the sum.

First method

$$\frac{3}{7}+\frac{5}{7}+\frac{-3}{7}+\frac{-5}{7}$$

$$=\left(\frac{3}{7}+\frac{-3}{7}\right)+\left(\frac{5}{7}+\frac{-5}{7}\right)$$

$$= 0+0=0$$

Sum of a number and its additive inverse is zero.

Second method

$$\left(\frac{3}{7}+\frac{5}{7}\right)+\left(\frac{-3}{7}+\frac{-5}{7}\right)$$

$$=\left(\frac{3+5}{7}\right)+\left(\frac{-3+(-5)}{7}\right)$$

$$=\frac{8}{7}+\left(\frac{-3-5}{7}\right)$$

$$=\frac{8}{7}+\left(\frac{-8}{7}\right)=0$$

Observe that the first method is simpler than the second.

Example 6: Find the value of $\frac{3}{5}+\frac{5}{4}+\frac{-7}{15}+\frac{-3}{8}$

Solution: $\frac{3}{5}+\frac{5}{4}+\frac{-7}{15}+\frac{-3}{8}$

$$=\left(\frac{3}{5}+\frac{-7}{15}\right)+\left(\frac{5}{4}+\frac{-3}{8}\right)$$ (This grouping simplifies the calculation.)

$$=\left(\frac{3\times 3}{5\times 3}+\frac{-7}{15}\right)+\left(\frac{5\times 2}{4\times 2}+\frac{-3}{8}\right)=\left(\frac{9}{15}+\frac{-7}{15}\right)+\left(\frac{10}{8}+\frac{-3}{8}\right)$$

$$=\frac{9+(-7)}{15}+\frac{10+(-3)}{8}=\frac{9-7}{15}+\frac{10-3}{8}=\frac{2}{15}+\frac{7}{8}$$

$$=\frac{2\times 8}{15\times 8}+\frac{7\times 15}{8\times 15}=\frac{16}{120}+\frac{105}{120}=\frac{16+105}{120}=\frac{121}{120}$$

Now, if we change the grouping.

$$\left(\frac{3}{5}+\frac{5}{4}\right)+\left(\frac{-7}{15}+\frac{-3}{8}\right)$$

$$=\left(\frac{3\times4}{5\times4}+\frac{5\times5}{4\times5}\right)+\left(\frac{-7\times8}{15\times8}+\frac{-3\times15}{8\times15}\right)$$

$$=\left(\frac{12}{20}+\frac{25}{20}\right)+\left(\frac{-56}{120}+\frac{-45}{120}\right)$$

$$=\left(\frac{12+25}{20}\right)+\left(\frac{-56+(-45)}{120}\right)$$

$$=\frac{37}{20}+\left(\frac{-56-45}{120}\right)=\frac{37}{20}+\frac{(-101)}{120}$$

$$=\frac{37\times6}{20\times6}+\frac{(-101)}{120}=\frac{222}{120}+\frac{(-101)}{120}$$

$$=\frac{222-101}{120}=\frac{121}{120}.$$

(This grouping makes the calculations longer and slightly difficult than the first grouping.)

Worksheet 1

1. Add the following:

(i) $\frac{2}{7}+\frac{5}{7}$ (ii) $\frac{-5}{9}+\frac{7}{9}$ (iii) $2\frac{3}{11}+\frac{9}{-11}$

(iv) $\frac{-5}{4}+\frac{5}{4}$ (v) $-2\frac{5}{6}+\frac{13}{-6}$ (vi) $\frac{-21}{3}+\frac{18}{3}$

2. Find the values of:

(i) $\frac{4}{9}+\frac{7}{4}$ (ii) $\frac{-7}{11}+\frac{1}{4}$ (iii) $\frac{5}{8}+\frac{-3}{5}$

(iv) $\frac{10}{63}+\frac{6}{7}$ (v) $\frac{-7}{64}+\frac{3}{-16}$ (vi) $\frac{5}{12}+\frac{-9}{20}$

3. Verify x + y = y + x for following values of x and y.

(i) $x=\frac{5}{7}, y=\frac{-3}{2}$ (ii) $x=5, y=\frac{3}{2}$

(iii) $x=\frac{-5}{14}, y=\frac{-1}{21}$ (iv) $x=-8, y=\frac{9}{2}$

4. **Verify x + (y + z) = (x + y) + z for following values of x, y and z.**

(i) $x = \frac{3}{4}, y = \frac{5}{6}, z = \frac{-7}{8}$

(ii) $x = \frac{2}{3}, y = \frac{-5}{6}, z = \frac{-7}{9}$

(iii) $x = \frac{3}{5}, y = \frac{-6}{9}, z = \frac{2}{10}$

(iv) $x = \frac{-3}{5}, y = \frac{-7}{10}, z = \frac{-8}{15}$

5. **Simplify.**

(i) $\frac{-3}{10} + \frac{12}{-10} + \frac{14}{10}$

(ii) $\frac{-5}{10} + \frac{6}{13} + 8$

(iii) $\frac{-5}{10} + \frac{9}{7} + \frac{3}{20} + \frac{-11}{14}$

(iv) $\frac{5}{36} + \frac{-7}{8} + \frac{6}{-72} + \frac{-3}{-12}$

6. **For $x = \frac{1}{5}$ and $y = \frac{3}{7}$, verify that $-(x + y) = (-x) + (-y)$.**

7. **Write True or False for the following statements.**

(i) $\frac{-2}{-3}$ is the additive inverse of $\frac{2}{3}$.

(ii) $\frac{2}{3} + \frac{4}{5}$ is a rational number.

(iii) $\frac{-5}{3} + \frac{5}{-3}$ is equal to zero.

(iv) 1 is the identity element of addition.

(v) $\frac{-9}{7} + 0 = 0$.

(vi) Additive inverse of $\frac{-3}{5}$ is $\frac{3}{5}$.

(vii) Negative of a negative rational number is negative.

SUBTRACTION OF RATIONAL NUMBERS

Remember

Subtracting y from x is same as adding the additive inverse of y to x, i.e. $x - y = x + (-y)$

Let us explain it with the help of some illustrations.

Example 7: Subtract $\frac{7}{5}$ from $\frac{6}{3}$.

Solution: $\frac{6}{3} - \frac{7}{5}$

$= \frac{6}{3} + \left(\frac{-7}{5}\right)$ ← Adding $\frac{6}{3}$ and additive inverse of $\frac{7}{5}$.

$= \frac{6 \times 5}{3 \times 5} + \frac{(-7) \times 3}{5 \times 3}$

$= \frac{30}{15} + \frac{(-21)}{15} = \frac{30 + (-21)}{15}$

$= \frac{30 - 21}{15} = \frac{9}{15} = \frac{3}{5}$ ← Standard form

Example 8: Subtract $\frac{5}{63}$ from $\frac{-6}{7}$.

Solution: $\frac{-6}{7} - \frac{5}{63} = \frac{-6 \times 9}{7 \times 9} - \frac{54}{63}$

$= \frac{-54}{63} - \frac{5}{63} = \frac{-54 - 5}{63} = \frac{-59}{63}$.

Note:

From above examples, it is clear that difference of two rational numbers is a rational number.

PROPERTIES OF SUBTRACTION OF RATIONAL NUMBERS

1. **Verify that the difference of two rational numbers, $\frac{2}{3}$ and $\frac{7}{6}$ does not remain same if the order of numbers is changed.**

$\frac{2}{3} - \frac{7}{6}$	$\frac{7}{6} - \frac{2}{3}$
$= \frac{2 \times 2}{3 \times 2} - \frac{7}{6}$	$= \frac{7}{6} - \frac{2 \times 2}{3 \times 2}$

$$= \frac{4}{6} - \frac{7}{6} \qquad\qquad = \frac{7}{6} - \frac{4}{6}$$

$$= \frac{4-7}{6} \qquad\qquad = \frac{7-4}{6}$$

$$= \frac{-3}{6} \qquad\qquad = \frac{3}{6}$$

Not Same

Property 1: **For rational numbers x and y, $x - y \neq y - x$ in general. In fact, $x - y = -(y - x)$, i.e. commutative property does not hold true for subtraction.**

2. Let us now observe the following cases.

(Grouping is changed)

$\left(\frac{5}{4} - \frac{3}{2}\right) - \frac{2}{3}$	$\frac{5}{4} - \left(\frac{3}{2} - \frac{2}{3}\right)$
$= \left(\frac{5}{4} - \frac{3\times 2}{2\times 2}\right) - \frac{2}{3}$	$= \frac{5}{4} - \left(\frac{3\times 3}{2\times 3} - \frac{2\times 2}{3\times 2}\right)$
$= \left(\frac{5}{4} - \frac{6}{4}\right) - \frac{2}{3}$	$= \frac{5}{4} - \left(\frac{9}{6} - \frac{4}{6}\right)$
$= \frac{5-6}{4} - \frac{2}{3}$	$= \frac{5}{4} - \left(\frac{9-4}{6}\right)$
$= \frac{-1}{4} - \frac{2}{3}$	$= \frac{5}{4} - \frac{5}{6}$
$= \frac{-1\times 3}{4\times 3} - \frac{2\times 4}{3\times 4}$	$= \frac{5\times 3}{4\times 3} - \frac{5\times 2}{6\times 2}$
$= \frac{-3}{12} - \frac{8}{12}$	$= \frac{15}{12} - \frac{10}{12}$
$= \frac{-3-8}{12}$	$= \frac{15-10}{12}$
$= \frac{-11}{12}$	$= \frac{5}{12}$

Not Same

Property 2: **For rational numbers x, y, z, in general, $(x - y) - z \neq x - (y - z)$, i.e. the associative property does not hold true for subtraction.**

3. For all rational numbers x, we have

$$x - 0 = x$$

but $\quad 0 - x = -x$

Therefore,

Property 3: **Identity element for subtraction does not exist.**

Property 4: **Since the identity element for subtraction does not exist, the question for finding inverse for subtraction does not arise.**

Following are a few examples to illustrate these properties.

Example 9: What number should be added to $\frac{-3}{5}$ so as to get $\frac{3}{7}$?

Solution: The required number will be obtained by subtracting $\frac{-3}{5}$ from $\frac{3}{7}$.

Therefore, required number shall be $= \frac{3}{7} - \left(\frac{-3}{5}\right) = \frac{3}{7} + \frac{3}{5}$

$$= \frac{3 \times 5}{7 \times 5} + \frac{3 \times 7}{5 \times 7} = \frac{15}{35} + \frac{21}{35}$$

$$= \frac{15 + 21}{35} = \frac{36}{35}$$

Example 10: Simplify: (i) $\frac{5}{4} - \frac{7}{6} - \left(\frac{-2}{3}\right)$ (ii) $\frac{-3}{5} - \left(\frac{-4}{15}\right) - \left(\frac{7}{-10}\right)$

Solution: (i) $\frac{5}{4} - \frac{7}{6} - \left(\frac{-2}{3}\right)$

2 has come by dividing 12 by 6

3 has come by dividing 12 by 4

4 has come by dividing 12 by 3

$$\frac{5 \times 3 - 7 \times 2 - (-2) \times 4}{12}$$ (LCM of 4, 6 and 3 is 12)

$$= \frac{15 - 14 - (-8)}{12} = \frac{15 - 14 + 8}{12}$$

$$= \frac{1 + 8}{12} = \frac{9}{12} = \frac{3}{4}$$ (standard form)

(ii) $\frac{-3}{5}-\left(\frac{-4}{15}\right)-\left(\frac{7}{-10}\right)$

First check whether all rational numbers are in standard form. Here $\frac{7}{-10}$ is not in standard form. Standard form of $\frac{7}{-10}$ is $\frac{-7}{10}$.

Now, we have $\frac{-3}{5}-\left(\frac{-4}{15}\right)-\left(\frac{-7}{10}\right)$

$$= \frac{(-3)\times 6-(-4)\times 2-(-7)\times 3}{30} \qquad \text{(LCM of 5, 10, 15 is 30)}$$

$$= \frac{-18-(-8)-(-21)}{30}$$

$$= \frac{-18+8+21}{30} = \frac{-10+21}{30} = \frac{11}{30}.$$

Worksheet 2

1. **Find the value of—**

   (i) $\frac{6}{7}-\frac{-5}{7}$ (ii) $\frac{5}{24}-\frac{7}{36}$

   (iii) $\frac{9}{10}-\frac{7}{-15}$ (iv) $\frac{-3}{8}-\frac{(-6)}{20}$

2. **Subtract.**

   (i) $\frac{5}{9}$ from $\frac{-7}{9}$ (ii) $\frac{-5}{7}$ from 0

   (iii) $\frac{5}{11}$ from $\frac{-8}{23}$ (iv) $\frac{-2}{9}$ from $\frac{7}{6}$

3. **The sum of two rational numbers is – 5. If one of the number is $\frac{2}{3}$, find the other.**

4. **What number should be added to $\frac{-3}{7}$ so as to get 1?**

5. **What number should be subtracted from – 1 so as to get $\frac{5}{3}$?**

6. **Simplify.**

(i) $\dfrac{-4}{5} - \dfrac{3}{15} + \dfrac{7}{20}$ (ii) $\dfrac{-5}{13} - \dfrac{-3}{26} - \dfrac{9}{-52}$

(iii) $\dfrac{7}{24} + \dfrac{5}{12} - \dfrac{11}{18}$ (iv) $\dfrac{-11}{30} - \dfrac{8}{15} + \dfrac{7}{6} + \dfrac{-2}{5}$

7. **Find the values of x – y and y – x for x = $\dfrac{2}{3}$, y = $\dfrac{5}{9}$. Are they equal?**

8. **For x = $\dfrac{1}{10}$, y = $\dfrac{-3}{5}$, z = $\dfrac{7}{20}$, find the values of the expressions (x – y) – z and x – (y – z). Are they equal?**

MULTIPLICATION OF RATIONAL NUMBERS

You have learnt in Class-V how to multiply two fractions. Let us recall.

$$\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15} \begin{array}{l} \leftarrow \text{Multiplication of numerators} \\ \leftarrow \text{Multiplication of denominators} \end{array}$$

In the same manner, we multiply the rational numbers.
For example,

$$\frac{-2}{3} \times \frac{4}{5} = \frac{(-2) \times 4}{3 \times 5} = \frac{-8}{15} \begin{array}{l} \leftarrow \text{Multiplication of numerators} \\ \leftarrow \text{Multiplication of denominators} \end{array}$$

Remember

If $\dfrac{a}{b}$ and $\dfrac{c}{d}$ are two rational numbers, then their product is given by,

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

$$\frac{\text{Product of numerators}}{\text{Product of denominators}}$$

Example 11: Multiply $\dfrac{4}{5}$ by $\dfrac{-10}{3}$.

Solution: $\dfrac{4}{5} \times \dfrac{-10}{3} = \dfrac{4 \times (-10)}{5 \times 3}$ (Divide (–10) and 5 by 5 to get rational number in standard form)

$$= \frac{4 \times (-2)}{3} = \frac{-8}{3}$$

Example 12: Find the product of $\frac{-6}{11}$ and $\frac{-7}{9}$.

Solution:

$$\frac{-6}{11} \times \frac{-7}{9} = \frac{(-6) \times (-7)}{11 \times 9} \quad \leftarrow \text{Not in the standard form}$$

$$= \frac{(-2) \times (-7)}{11 \times 3} \quad \leftarrow \text{Dividing } -6 \text{ and } 9 \text{ by their common factor } 3$$

$$= \frac{14}{33}$$

Note:

It is clear from above examples that the product of two rational numbers is a rational number.

PROPERTIES OF MULTIPLICATION OF RATIONAL NUMBERS

Multiplication of rational numbers has properties similar to those of multiplication of fractions.

1. Verify that the product of two rational numbers $\frac{3}{5}$ and $\frac{(-4)}{7}$ remains the same even if the order is changed.

$$\frac{3}{5} \times \frac{(-4)}{7} = \frac{3 \times (-4)}{5 \times 7} = \frac{-12}{35}$$

$$\frac{(-4)}{7} \times \frac{3}{5} = \frac{(-4) \times 3}{7 \times 5} = \frac{-12}{35}$$

Product is same

Property 1: **Product of two rational numbers remains the same even if we change their order, i.e. if x and y are rational numbers, then**

$$x \times y = y \times x.$$

This is commutative law of multiplication.

2. Verify that the product of rational numbers $\frac{-3}{4}, \frac{5}{7}, \frac{-2}{11}$ remains the same even after changing the groupings.

$$\left(\frac{-3}{4} \times \frac{5}{7}\right) \times \left(\frac{-2}{11}\right) \qquad \frac{-3}{4} \times \left(\frac{5}{7} \times \frac{-2}{11}\right)$$

$$= \left(\frac{-3\times 5}{4\times 7}\right)\times\left(\frac{-2}{11}\right)$$

$$= \frac{-15}{28}\times\frac{(-2)}{11}$$

$$= \frac{(-15)\times(-1)}{14\times 11} \text{ (in lowest terms)}$$

$$= \frac{15}{154}$$

$$= \frac{-3}{4}\times\left(\frac{5\times(-2)}{7\times 11}\right)$$

$$= \frac{-3}{4}\times\frac{-10}{77}$$

$$= \frac{(-3)\times(-10)}{4\times 77} \text{ (in lowest terms)}$$

$$= \frac{(-3)\times(-5)}{2\times 77}$$

$$= \frac{15}{154}$$

Product is same

Property 2: **Product remains the same even when we change the grouping of the rational numbers, i.e. if x, y and z are rational numbers, then**

$$(x \times y) \times z = x \times (y \times z)$$

This is associative law of multiplication.

3. Find the product of $\frac{-4}{7}$ and 0.

Now, $\frac{-4}{7}\times 0 = \frac{-4\times 0}{7\times 1} = \frac{-0}{7} = 0$

Similarly, $0\times\frac{-4}{7} = \frac{0\times(-4)}{1\times 7} = \frac{0}{7} = 0$

Property 3: **Product of a rational number and zero is zero, i.e. if x is any rational number, then**

$$x \times 0 = 0 = 0 \times x$$

4. Find the product of $\frac{-15}{37}$ and 1.

Now, $\frac{-15}{37}\times 1 = \frac{-15}{37}\times\frac{1}{1} = \frac{-15\times 1}{37\times 1} = \frac{-15}{37}$

Similarly, $1\times\frac{-15}{37} = \frac{1}{1}\times\frac{-15}{37} = \frac{1\times -15}{37} = \frac{-15}{37}$

Property 4: **One multiplied by any rational number is the rational number itself, i.e. if x is a rational number, then**

$$x \times 1 = 1 \times x = x$$

i.e. 1 is the identity element under multiplication.

5. **Take three rational numbers, $\frac{2}{5}, \frac{5}{7}$ and $\frac{6}{15}$ and find out the value of $\frac{2}{5} \times \left(\frac{5}{7} + \frac{6}{15}\right)$ and $\left(\frac{2}{5} \times \frac{5}{7}\right) + \left(\frac{2}{5} \times \frac{6}{15}\right)$.**

$$\frac{2}{5} \times \left(\frac{5}{7} + \frac{6}{15}\right)$$
$$= \frac{2}{5} \times \left(\frac{75+42}{105}\right)$$
$$= \frac{2}{5} \times \left(\frac{117}{105}\right)$$
$$= \frac{2}{5} \times \frac{39}{35} \quad \text{(in lowest terms)}$$
$$= \frac{78}{175} \quad \text{(in lowest terms)}$$

$$\left(\frac{2}{5} \times \frac{5}{7}\right) + \left(\frac{2}{5} \times \frac{6}{15}\right)$$
$$= \frac{2 \times 5}{5 \times 7} + \frac{2 \times 6}{5 \times 15}$$
$$= \frac{2}{7} + \frac{2 \times 2}{5 \times 5} \quad \text{(in lowest terms)}$$
$$= \frac{2}{7} + \frac{4}{25}$$
$$= \frac{50+28}{175} = \frac{78}{175}$$

In both cases, the result is the same.

So, we can say $\frac{2}{5} \times \left(\frac{5}{7} + \frac{6}{15}\right) = \left(\frac{2}{5} \times \frac{5}{7}\right) + \left(\frac{2}{5} \times \frac{6}{15}\right)$

Property 5: **If x, y and z are rational numbers, then**

(i) $x \times (y + z) = x \times y + x \times z$ (ii) $x \times (y - z) = x \times y - x \times z$

This is distributive law of multiplication over addition.

We illustrate (ii) with the help of the following example.

Example 13: For rational numbers $x = \frac{5}{6}$, $y = \frac{-7}{3}$, $z = \frac{2}{9}$, verify that $x \times (y - z) = x \times y - x \times z$.

Solution:

$$x \times (y - z)$$
$$= \frac{5}{6} \times \left[\frac{-7}{3} - \frac{2}{9}\right]$$
$$= \frac{5}{6} \times \left[\frac{-7 \times 3 - 2}{9}\right]$$
$$= \frac{5}{6} \times \left[\frac{-21-2}{9}\right]$$
$$= \frac{5}{6} \times \left[\frac{-23}{9}\right]$$

$$x \times y - x \times z$$
$$= \frac{5}{6} \times \frac{-7}{3} - \frac{5}{6} \times \frac{2}{9}$$
$$= \frac{5 \times -7}{6 \times 3} - \frac{5 \times 2}{6 \times 9}$$
$$= \frac{-35}{18} - \frac{10}{54}$$
$$= \frac{-35 \times 3 - 10}{54}$$

$= \dfrac{5 \times (-23)}{6 \times 9}$ | $= \dfrac{-105-10}{54}$

$= \dfrac{-115}{54}$ | $= \dfrac{-115}{54}$

Therefore, $x \times (y - z) = x \times y - x \times z$

Worksheet 3

1. Multiply and express the result as a rational number in the standard form.

(i) $\dfrac{11}{7}$ by $\dfrac{-3}{8}$ (ii) $\dfrac{-7}{4}$ by $\dfrac{2}{3}$ (iii) $\dfrac{-3}{12}$ by -48

(iv) $\dfrac{-14}{9}$ by $\dfrac{-3}{7}$ (v) $\dfrac{23}{5}$ by $\dfrac{-25}{11}$ (vi) 7 by $\dfrac{-15}{63}$

2. For the following values of x and y, verify x × y = y × x.

(i) $x = \dfrac{7}{9}, y = \dfrac{3}{2}$ (ii) $x = \dfrac{-2}{7}, y = \dfrac{5}{8}$

(iii) $x = \dfrac{4}{9}, y = \dfrac{-5}{11}$ (iv) $x = \dfrac{-17}{48}, y = \dfrac{-96}{51}$

3. For the following values of x, y and z, find the products (x × y) × z and x × (y × z) and observe the result (x × y) × z = x × (y × z).

(i) $x = \dfrac{3}{5}, y = \dfrac{-7}{3}, z = \dfrac{8}{11}$ (ii) $x = \dfrac{-7}{11}, y = \dfrac{4}{5}, z = \dfrac{3}{8}$

(iii) $x = \dfrac{-4}{7}, y = \dfrac{-3}{8}, z = \dfrac{16}{5}$ (iv) $x = -3, y = \dfrac{-4}{9}, z = \dfrac{-7}{3}$

4. Verify the property x × (y + z) = x × y + x × z by taking—

(i) $x = \dfrac{1}{3}, y = \dfrac{1}{5}, z = \dfrac{1}{7}$ (ii) $x = \dfrac{-3}{7}, y = \dfrac{2}{5}, z = \dfrac{-4}{9}$

5. Show that–

$$\frac{-4}{3} \times \left(\frac{2}{5} + \frac{-7}{10}\right) = \left(\frac{-4}{3} \times \frac{2}{5}\right) + \left(\frac{-4}{3} \times \frac{-7}{10}\right)$$

6. Show that–

$$\frac{3}{5} \times \left(\frac{-1}{7} - \frac{5}{14}\right) = \left(\frac{3}{5} \times \frac{-1}{7}\right) - \left(\frac{3}{5} \times \frac{5}{14}\right)$$

7. Simplify and express the result in standard form.

(i) $-4 \times \left(\frac{7}{3} - \frac{9}{10}\right)$

(ii) $\frac{7}{3} \times \left(\frac{9}{8} + 3\right)$

(iii) $\left(\frac{-4}{3} + \frac{5}{7}\right) \times \frac{7}{9}$

(iv) $\left(\frac{5}{4} - \frac{6}{20}\right) \times \frac{8}{11}$

8. Fill in the blanks.

(i) $\frac{-4}{7} \times$ _____ $= \frac{-4}{7}$

(ii) $\frac{3}{8} \times$ _____ $= \frac{-3}{8}$

(iii) $\left(\frac{-1}{3} \times \frac{4}{5}\right) \times \frac{6}{7} =$ _____ $\times \left(\frac{4}{5} \times \frac{6}{7}\right)$

(iv) $\frac{5}{3} \times \left(\frac{-7}{8} \times \frac{11}{3}\right) = \left(\frac{5}{3} \times\right.$ _____ $\left.\right) \times \frac{11}{3}$

(v) $\frac{3}{7} \times \frac{-6}{11} = \frac{-6}{11} \times$ _____

(vi) $\frac{4}{3} \times$ _____ $= 0$

(vii) For any rational number x, x × 5 = x + x + ... _____ times.

(viii) $\frac{2}{3} \times \left(\frac{7}{5} - \frac{2}{9}\right) = \frac{2}{3} \times \frac{7}{5} -$ _____

(ix) $\frac{-5}{7} \times \frac{1}{3} + \frac{-5}{7} \times \frac{1}{6} = \frac{-5}{7} \times \left(\frac{1}{3} +\right.$ _____ $\left.\right)$

(x) $\frac{4}{7} \times \frac{2}{3} - \frac{4}{7} \times \frac{5}{6} = \frac{4}{7} \times ($ _____ $-$ _____ $)$

RECIPROCAL OF A RATIONAL NUMBER

Let us consider a rational number $\frac{9}{4}$. We try to find a rational number which when multiplied by $\frac{9}{4}$ gives us the result 1. In other words, let us try to fill in the blanks so that the statement $\frac{9}{4} \times$ _____ $= \frac{1}{1} = 1$.

You must fill in the blank by $\frac{4}{9}$ so that

$$\frac{9}{4} \times \frac{4}{9} = \frac{36}{36} = \frac{1}{1} = 1.$$

The product of $\frac{9}{4}$ and $\frac{4}{9}$ is 1.

$\frac{4}{9}$ is called the **reciprocal (multiplicative inverse)** of $\frac{9}{4}$.

Verify whether (i) $\frac{-4}{3}$ is reciprocal of $\frac{-3}{4}$ (ii) $\frac{-7}{3}$ is reciprocal of $\frac{3}{7}$.

(i) Find the product.

$$\frac{-4}{3} \times \frac{-3}{4} = \frac{(-4) \times (-3)}{3 \times 4} = \frac{12}{12} = 1$$

Hence, $\frac{-3}{4}$ is the reciprocal of $\frac{-4}{3}$.

(ii) Observe the product.

$$\frac{3}{7} \times \frac{-7}{3} = \frac{3 \times (-7)}{7 \times 3} = \frac{-21}{21} = -1$$

Since we are getting – 1, therefore, $\frac{-7}{3}$ is not the reciprocal of $\frac{3}{7}$.

Two non-zero numbers $\frac{a}{b}$ and $\frac{b}{a}$ are reciprocals of each other.

Note:

(i) To get the reciprocal of a given rational number, simply interchange the integers in the numerator and the denominator.

(ii) Zero has no reciprocal.

(iii) Reciprocal of 1 is 1.

(iv) If x is any non-zero rational number, then its reciprocal is denoted by x^{-1} which is equal to $\frac{1}{x}$.

Example 14: Find the reciprocals of the following rational numbers.

(i) $\frac{-3}{7}$ (ii) $\frac{3}{-7}$ (iii) $\frac{-3}{-7}$

Solution: By simply interchanging the numerator and denominator, we can find reciprocal.

(i) Reciprocal of $\frac{-3}{7}$ is $\frac{7}{-3}$.

(ii) Reciprocal of $\frac{3}{-7}$ is $\frac{-7}{3}$.

(iii) Reciprocal of $\frac{-3}{-7}$ is $\frac{-7}{-3}$.

Example 15: Verify $(x \times y)^{-1} = x^{-1} \times y^{-1}$, for $x = \frac{-2}{5}$, $y = \frac{3}{5}$.

Solution: $(x \times y)^{-1} = \left(\frac{-2}{5} \times \frac{3}{5}\right)^{-1} = \left(\frac{-2 \times 3}{5 \times 5}\right)^{-1} = \left(\frac{-6}{25}\right)^{-1} = \frac{25}{-6}$

Now, $x^{-1} \times y^{-1} = \frac{5}{-2} \times \frac{5}{3} = \frac{5 \times 5}{-2 \times 3} = \frac{26}{-6}$.

In both the cases the value is the same.

Hence, $(x \times y)^{-1} = x^{-1} \times y^{-1}$.

Example 16: Check the validity of the result, $(x + y)^{-1} \neq x^{-1} + y^{-1}$ for $x = \frac{1}{3}$, $y = \frac{-2}{7}$.

Solution:

$$(x + y)^{-1} = \left(\frac{1}{3} + \frac{-2}{7}\right)^{-1}$$

$$= \left[\frac{7 + (-6)}{21}\right]^{-1} = \left[\frac{7-6}{21}\right]^{-1} = \left[\frac{1}{21}\right]^{-1} = 21$$

Now, $x^{-1} + y^{-1} = \left(\frac{1}{3}\right)^{-1} + \left(\frac{-2}{7}\right)^{-1} = 3 + \frac{7}{-2}$

$$= \frac{3}{1} + \frac{-7}{2}$$

$$= \frac{6-7}{2} = \frac{-1}{2}$$

Hence, $(x + y)^{-1} \neq x^{-1} + y^{-1}$.

Worksheet 4

1. **Find the reciprocals of:**

(i) $\frac{-1}{5}$ (ii) 4 (iii) $\frac{11}{-12}$ (iv) $\frac{-2}{-19}$

2. **Check, if the reciprocal of $\frac{-2}{3}$ is $\frac{3}{2}$?**

3. **Check if the reciprocal of $\frac{-1}{5}$ is -5?**

4. **Verify that $(x - y)^{-1} \neq x^{-1} - y^{-1}$ by taking $x = \frac{-2}{7}$, $y = \frac{4}{7}$.**

   [Hint: Proceed as in Example 16.]

5. **Verify that $(x + y)^{-1} \neq x^{-1} + y^{-1}$ by taking $x = \frac{5}{9}$ and $y = \frac{-4}{3}$.**

6. **Verify that $(x \times y)^{-1} = x^{-1} \times y^{-1}$ by taking $x = \frac{-2}{3}$ and $y = \frac{-3}{4}$.**

7. **Fill in the blanks.**

   (i) The number _________ has no reciprocal.

   (ii) _________ and _________ are their own reciprocals.

   (iii) If a is the reciprocal of b, then b is the reciprocal of _________.

   (iv) $(11 \times 5)^{-1} = (11)^{-1} \times$ _________.

   (v) $\frac{-1}{8} \times$ _________ $= 1$

   (vi) _________ $\times \left(-5\frac{1}{3}\right) = 1$

DIVISION OF RATIONAL NUMBERS

Remember

Dividing one rational number by another except by zero, is the same as the multiplication of the first by the reciprocal of the second,

i.e. $x \div y = x \times y^{-1}$

We illustrate this with the help of examples.

Example 17: Divide $\frac{7}{3}$ by $\frac{5}{3}$.

Solution:

$$\frac{7}{3} \div \frac{5}{3} = \frac{7}{3} \times \left(\frac{5}{3}\right)^{-1}$$

$$= \frac{7}{3} \times \frac{3}{5} = \frac{7}{5}.$$

Example 18: Divide $\frac{2}{9}$ by – 4.

Solution:

$$\frac{2}{9} \div (-4) = \frac{2}{9} \times (-4)^{-1}$$

$$= \frac{2}{9} \times \frac{1}{-4}$$

$$= \frac{-1}{18} \quad \text{(Standard form)}$$

Example 19: The product of two rational numbers is $\frac{-25}{16}$. If one of the numbers is $\frac{-5}{4}$, find the other.

Solution: Product of two numbers = $\frac{-25}{16}$

One number = $\frac{-5}{4}$

We can write it as $\frac{-5}{4} \times \boxed{\text{other number}} = \frac{-25}{16}$

or other number $= \frac{-25}{16} \div \frac{-5}{4} = \frac{-25}{16} \times \left(\frac{-5}{4}\right)^{-1}$

$$= \frac{-25}{16} \times \frac{4}{-5}$$

$$= \frac{5 \times 1}{4 \times 1} = \frac{5}{4}$$

PROPERTIES OF DIVISION OF RATIONAL NUMBERS

1. Divide two rational numbers and find the result.

(i) $\frac{5}{3} \div \frac{-4}{3} = \frac{5}{3} \times \frac{3}{-4} = \frac{-5}{4}$

(ii) $\frac{4}{9} \div \frac{9}{4} = \frac{4}{9} \times \frac{4}{9} = \frac{16}{81}$ Rational Numbers

Similarly, (iii) $0 \div \frac{-4}{5} = 0$

Property 1: **Division of a rational number by another rational number except zero, is a rational number.**

2. **Find the quotient, if rational number $\frac{-3}{4}$ is divided by the same rational number.**

$$\frac{-3}{4} \div \frac{-3}{4} = \frac{-3}{4} \times \frac{4}{-3}$$

$$= 1$$

Property 2: **When a rational number (non-zero) is divided by the same rational number, the quotient is one.**

3. **Find the quotient when a rational number $\frac{-4}{5}$ is divided by 1.**

$$\frac{-4}{5} \div 1 = \frac{-4}{5} \times \frac{1}{1}$$

$$= \frac{-4}{5}$$

Property 3: **When a rational number is divided by 1, the quotient is the same rational number.**

4. **If $x = \frac{3}{2}$, $y = \frac{-4}{5}$, prove that $x \div y \neq y \div x$.**

$x \div y$	$y \div x$
$= \frac{3}{2} \div \frac{-4}{5}$	$= \frac{-4}{5} \div \frac{3}{2}$
$= \frac{3}{2} \times \frac{5}{-4}$	$= \frac{-4}{5} \times \frac{2}{3}$
$= \frac{15}{-8}$	$= \frac{-8}{15}$
$= \frac{-15}{8}$	

Not the same

Therefore, $x \div y \neq y \div x$.

Property 4: **If x and y are non-zero rational numbers, then in general, $x \div y \neq y \div x$ i.e. commutative property does not hold true for division.**

Note:

In fact $x \div y = \frac{1}{y \div x}$

5. If $x = \frac{2}{3}$, $y = \frac{-4}{9}$, $z = \frac{5}{6}$, prove that $(x \div y) \div z \neq x \div (y \div z)$.

$(x \div y) \div z$

$$= \left(\frac{2}{3} \div \frac{-4}{9}\right) \div \frac{5}{6}$$

$$= \left(\frac{2}{3} \times \frac{9}{-4}\right) \div \frac{5}{6}$$

$$= \frac{-3}{2} \div \frac{5}{6}$$

$$= \frac{-3}{2} \times \frac{6}{5}$$

$$= \frac{-3 \times 3}{5}$$

$$= \frac{-9}{5}$$

$x \div (y \div z)$

$$= \frac{2}{3} \div \left(\frac{-4}{9} \div \frac{5}{6}\right)$$

$$= \frac{2}{3} \div \left(\frac{-4}{9} \times \frac{6}{5}\right)$$

$$= \frac{2}{3} \div \left(\frac{-4 \times 2}{3 \times 5}\right)$$

$$= \frac{2}{3} \div \left(\frac{-8}{15}\right)$$

$$= \frac{2}{3} \times \frac{15}{-8}$$

$$= \frac{5}{-4} = \frac{-5}{4}$$

Not the same

Hence, $(x \div y) \div z \neq x \div (y \div z)$

Property 5: **If x, y and z are non-zero rational numbers, then, in general**

$$(x \div y) \div z \neq x \div (y \div z),$$

i.e. associative property does not hold true for division.

6. Take $x = \frac{-2}{3}$, $y = \frac{5}{9}$, $z = \frac{-1}{6}$ and prove that $(x + y) \div z = x \div z + y \div z$ and $(x - y) \div z = x \div z - y \div z$.

(i) First case

$(x + y) \div z$

$$= \left(\frac{-2}{3} + \frac{5}{9}\right) \div \frac{(-1)}{6}$$

$$= \left(\frac{-6+5}{9}\right) \div \frac{(-1)}{6}$$

$$= \frac{-1}{9} \div \frac{(-1)}{6}$$

$$= \frac{-1}{9} \times \frac{6}{-1} = \frac{2}{3}$$

$(x \div z) + (y \div z)$

$$= \frac{-2}{3} \div \frac{(-1)}{6} + \frac{5}{9} \div \frac{(-1)}{6}$$

$$= \frac{-2}{3} \times \frac{6}{(-1)} + \frac{5}{9} \times \frac{6}{(-1)}$$

$$= 4 - \frac{5 \times 2}{3} = \frac{4}{1} - \frac{10}{3}$$

$$= \frac{12 - 10}{3} = \frac{2}{3}$$

Thus, we have verified that $(x + y) \div z = x \div z + y \div z$.

(ii) Second case

$(x-y) \div z$	$x \div z - y \div z$
$= \left(\frac{-2}{3} - \frac{5}{9}\right) \div \frac{(-1)}{6}$	$= \frac{-2}{3} \div \frac{(-1)}{6} - \frac{5}{9} \div \frac{(-1)}{6}$
$= \left(\frac{-6-5}{9}\right) \div \frac{(-1)}{6}$	$= \frac{-2}{3} \times \frac{6}{(-1)} - \frac{5}{9} \times \frac{6}{(-1)}$
$= \frac{-11}{9} \times \frac{6}{(-1)}$	$= 2 \times 2 + \frac{5 \times 2}{3} = \frac{4}{1} + \frac{10}{3}$
$= \frac{11 \times 2}{3} = \frac{22}{3}$	$= \frac{12+10}{3} = \frac{22}{3}$

Here again, we have verified that $(x - y) \div z = x \div z - y \div z$.

Property 6: **If x, y and z are rational numbers, then $(x + y) \div z = x \div z + y \div z$ and $(x - y) \div z = x \div z - y \div z$**

7. If $x = \frac{2}{5}$, $y = \frac{-3}{10}$, $z = \frac{4}{15}$, prove $x \div (y + z) \neq x \div y + x \div z$.

$x \div (y+z)$	$x \div y + x \div z$
$= \frac{2}{5} \div \left(\frac{-3}{10} + \frac{4}{15}\right)$	$= \frac{2}{5} \div \frac{-3}{10} + \frac{2}{5} \div \frac{4}{15}$
$= \frac{2}{5} \div \left(\frac{-9+8}{30}\right)$	$= \frac{2}{5} \times \frac{10}{-3} + \frac{2}{5} \times \frac{15}{4}$
$= \frac{2}{5} \div \frac{-1}{30}$	$= \frac{-2 \times 2}{3} + \frac{1 \times 3}{1 \times 2}$
$= \frac{2}{5} \times \frac{30}{-1}$	$= \frac{-4}{3} + \frac{3}{2}$
$= -2 \times 6$	$= \frac{-8+9}{6}$
$= -12$	$= \frac{1}{6}$

Not the same

Hence, $x \div (y + z) \neq x \div y + x \div z$

Similarly, we may verify that $x \div (y - z) \neq x \div y - x \div z$.

Property 7: **For three non-zero rational numbers x, y and z, $x \div (y + z) \neq x \div y + x \div z$, i.e. distributive property does not hold true for division.**

Worksheet 5

1. **Divide.**

   (i) $\frac{2}{5}$ by $\frac{-1}{3}$ (ii) $\frac{-7}{4}$ by $\frac{1}{8}$ (iii) -10 by $\frac{1}{5}$ (iv) $\frac{1}{13}$ by -2

2. **By taking $x = \frac{3}{4}$ and $y = \frac{-5}{6}$, verify that $x \div y \neq y \div x$.**

3. **The product of two rational numbers is $\frac{-3}{7}$. If one of the number is $\frac{5}{21}$, find the other.**

4. **With what number should we multiply $\frac{-36}{35}$, so that the product be $\frac{-6}{5}$?**

5. **By taking $x = \frac{-5}{3}$, $y = \frac{2}{7}$ and $z = \frac{1}{-4}$, verify that–**

   (i) $x \div (y + z) \neq x \div y + x \div z$

   (ii) $x \div (x - z) \neq x \div y - x \div z$

   (iii) $(x + y) \div z = x \div z + y \div z$

6. **From a rope of the length 40 metres, a man cuts some equal sized pieces. How many pieces can be cut if each piece is of $\frac{4}{9}$ metres length?**

RATIONALS BETWEEN TWO RATIONAL NUMBERS

Let us find a rational number between $\frac{1}{4}$ and $\frac{3}{4}$.

For getting one rational number between $\frac{1}{4}$ and $\frac{3}{4}$, we add the two given rational numbers and then divide the sum by 2. That is,

$$\frac{1}{2}\left(\frac{1}{4}+\frac{3}{4}\right) = \frac{1}{2}\left(\frac{1+3}{4}\right)$$

$$= \frac{1}{2}\left(\frac{4}{4}\right) = \frac{1}{2}$$

which is a rational number lying between $\frac{1}{4}$ and $\frac{3}{4}$.

If x and y are two rational numbers, then $\frac{x+y}{2}$ is a rational number between x and y.

Example 20: Find three rational numbers between $\frac{1}{2}$ and $\frac{-1}{2}$.

Solution: **Step 1:** Find a rational number between $\frac{1}{2}$ and $\frac{-1}{2}$.

$$\frac{1}{2}\left[\frac{1}{2}+\left(\frac{-1}{2}\right)\right]=\frac{1}{2}\times 0=0$$

Step 2: Find a rational number between $\frac{1}{2}$ and 0.

$$\frac{1}{2}\left[\frac{1}{2}+0\right]=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$$

Step 3: Find a rational number between $\left(\frac{-1}{2}\right)$ and 0.

$$\frac{1}{2}\left[\frac{-1}{2}+0\right]=\frac{1}{2}\times\left(\frac{-1}{2}\right)=\frac{-1}{4}$$

Hence, 0, $\frac{1}{4}$, $\frac{-1}{4}$ are three rational numbers lying between $\frac{1}{2}$ and $\frac{-1}{2}$.

Note:

Between any two rational numbers, there are infinitely many rational numbers.

Worksheet 6

1. **Correct the following statements.**
   (i) Between two rational numbers, we can find only one rational number.
   (ii) Between two rational numbers, we can find as many integers as we like.
   (iii) Between two integers, we can find as many integers as we like.

2. **Find a rational number between:**

(i) 2 and 4

(ii) -2 and -6

(iii) $\frac{1}{4}$ and $\frac{-3}{4}$

(iv) $\frac{-2}{3}$ and $\frac{-7}{3}$

3. **Insert three rational numbers between:**

(i) $\frac{4}{13}$ and $\frac{1}{13}$

(ii) $\frac{-7}{10}$ and $\frac{11}{10}$

(iii) $\frac{-4}{3}$ and $\frac{-19}{3}$

(iv) $\frac{1}{8}$ and $\frac{-15}{8}$

4. **Find five rational numbers between:**

(i) $\frac{-4}{7}$ and $\left|\frac{-4}{7}\right|$

(ii) $\frac{-8}{3}$ and $\left|\frac{-8}{3}\right|$

VALUE BASED QUESTION

Rohit donated $\frac{1}{5}$ of his monthly income to an Non-Government Organisation (NGO) working for the education of the girl child, spent $\frac{1}{4}$ of his salary on food, $\frac{1}{3}$ on rent and $\frac{1}{15}$ on other expenses. He is left with ₹ 9000.

(a) Find Rohit's monthly salary.

(b) What values of Rohit are depicted here?

(c) Why is the education, specially for girls, important?

BRAIN TEASERS

1. A. **Tick (✓) the correct option.**

(a) The additive inverse of $\frac{-3}{4}$ is–

(i) $\frac{-3}{4}$ (ii) $\frac{-4}{3}$ (iii) $\frac{4}{3}$ (iv) $\frac{3}{4}$

(b) If x, y and z are rational numbers, then the property $(x + y) + z = x + (y + z)$ is known as–

(i) commutative property (ii) associative property

(iii) distributive property (iv) closure property

(c) $\frac{7}{12} \div \left(\frac{-7}{12}\right)$ is–

(i) 1 (ii) 7 (iii) – 1 (iv) – 7

(d) Identity element for subtraction of rational numbers is–

(i) 1 (ii) 0 (iii) – 1 (iv) does not exist

(e) The multiplicative inverse of $6\frac{1}{3}$ is–

(i) $\frac{-19}{3}$ (ii) $\frac{-3}{19}$ (iii) $\frac{3}{19}$ (iv) $\frac{19}{3}$

B. Answer the following questions.

(a) Write all rational numbers whose absolute value is $\frac{5}{9}$.

(b) Find the reciprocal of $\frac{4}{5} \times \left(\frac{3}{-8}\right)$.

(c) What should be added to $\frac{-5}{11}$ to get $\frac{26}{33}$?

(d) Subtract $6\frac{2}{3}$ from the sum of $\frac{-3}{7}$ and 2.

(e) Find the value of $1+\cfrac{1}{1+\cfrac{1}{6}}$.

2. State whether the following statements are true or false. If false, justify your answer with an example.

(i) If $|x| = 0$, then x has no reciprocal. __________

(ii) If $x < y$ then $|x| < |y|$. __________

(iii) If $x < y$ then $x^{-1} < y^{-1}$ __________

(iv) The negative of a negative rational number is a positive rational number. __________

(v) Product of two rational numbers can never be an integer. ____________

(vi) Product of two integers is never a fraction. ____________

(vii) If x and y are two rational numbers such that x > y, then x – y is always a positive rational number. ____________

3. For $x = \frac{3}{4}$ and $y = \frac{-9}{8}$, insert a rational number between:

(i) $(x + y)^{-1}$ and $x^{-1} + y^{-1}$ (ii) $(x - y)^{-1}$ and $x^{-1} - y^{-1}$.

4. Verify that–

$(x \div y)^{-1} = x^{-1} \div y^{-1}$ by taking $x = \frac{-5}{11}$, $y = \frac{7}{3}$.

5. Verify that $|x + y| \leq |x| + |y|$ by taking $x = \frac{2}{3}$, $y = \frac{-3}{5}$.

6. Find the reciprocals of:

(i) $\frac{2}{-5} \times \frac{3}{-7}$ (ii) $\frac{-4}{3} \times \frac{-5}{-8}$

7. Simplify.

(i) $\left|\frac{5}{7} - \frac{2}{3}\right| + \left|\frac{3}{14} - \frac{5}{7}\right|$ (ii) $\left(\frac{5}{11}\right)^{-1} - \frac{13}{5} + \frac{3}{15}$

(iii) $\frac{9}{5} \times \frac{-2}{27} + \frac{7}{30}$ (iv) $\frac{-7}{15} \div \left(\frac{50}{3}\right)^{-1}$

8. Divide.

(i) The sum of $\frac{5}{21}$ and $\frac{4}{7}$ by their difference.

(ii) The difference of $\frac{12}{5}$, $\frac{-16}{20}$ by their product.

9. Find reciprocal of $\frac{-2}{3} \times \frac{5}{7} + \frac{2}{9} \div \frac{1}{3} \times \frac{6}{7}$.

HOTS

1. A drum of kerosene oil is $\frac{3}{4}$ full. When 15 litres of oil is drawn from it, it is $\frac{7}{12}$ full. Find the total capacity of the drum.

2. **Find the product of:**

$$\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\ldots\left(1-\frac{1}{10}\right)$$

ENRICHMENT QUESTION

Complete the following magic square of multiplication.

$\frac{1}{81}\times\frac{1}{81}$		$\frac{1}{81}\times\frac{1}{9}$
	$\frac{1}{3}\times\frac{1}{81}$	
$\frac{1}{27}\times\frac{1}{3}$		$\frac{1}{3}\times\frac{1}{3}$

YOU MUST KNOW

1. If $\frac{a}{b}$ and $\frac{c}{d}$ are non-zero rational numbers, then–

   (i) $\frac{a}{b}+\frac{c}{d}=\frac{a\times d+b\times c}{b\times d}$

   (ii) $\frac{a}{b}-\frac{c}{d}=\frac{a\times d-b\times c}{b\times d}$

   (iii) $\frac{a}{b}\times\frac{c}{d}=\frac{a\times c}{b\times d}$

   (iv) $\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}=\frac{a\times d}{b\times c}$

2. If x, y and z are rational numbers, then–

   (i) $x + y$ is a rational number.

   (ii) $x + y = y + x$

   (iii) $0 + x = x + 0 = x$

   (iv) $x + (y + z) = (x + y) + z$

3. If x, y, and z are rational numbers, then–

   (i) $x - y$ is a rational number.

   (ii) $x - y \neq y - x$

   (iii) $x - 0 = x \neq 0 - x$

   (iv) $(x - y) - z \neq x - (y - z)$

4. If x, y, and z are rational numbers, then–

   (i) $x \times y$ is a rational number.
   (ii) $x \times y = y \times x$
   (iii) $x \times 0 = 0 = 0 \times x$
   (iv) $x \times 1 = x = 1 \times x$
   (v) $(x \times y) \times z = x \times (y \times z)$
   (vi) $x \times (y + z) = x \times y + x \times z$
   (vii) $x \times (y - z) = x \times y - x \times z$

5. Two non-zero rational numbers $\frac{a}{b}$ and $\frac{b}{a}$ are reciprocals of each other.

6. If x, y and z are rational numbers, then–

   (i) $x \div y$ is also a rational number, $y \neq 0$
   (ii) $x \div 1 = x$
   (iii) $x \div y \neq y \div x$ in general
   (iv) $(x \div y) \div z \neq x \div (y \div z)$
   (v) $x \div (y + z) \neq x \div y + x \div z$
   (vi) $x \div (y - z) \neq x \div y - x \div z$
   (vii) $(x + y) \div z = x \div z + y \div z$
   (viii) $(x - y) \div z = x \div z - y \div z$

7. (i) $\frac{x+y}{2}$ is a rational number between two rational numbers x and y.

   (ii) Between any two rational numbers, there are infinitely many rational numbers.

CHAPTER 3

RATIONAL NUMBERS AS DECIMALS

INTRODUCTION

Do you remember how to write fraction in decimal form?

$\frac{5}{10} = 0.5$ $\frac{13}{100} = 0.13$ $\frac{11}{10} = 1.1$

$\frac{7}{1000} = 0.007$ $\frac{231}{1000} = 0.231$

Any fraction having denominator as 10 or a power of 10 (i.e. 10, 100, 1000, . . .) can be easily represented in decimal form.

RATIONAL NUMBERS AS DECIMALS

Here, we shall study how to represent a rational number in decimal form, if the denominator is not 10 or a power of 10.

Let us do some examples.

Example 1: Convert $\frac{3}{4}$ in its decimal form.

Solution: Convert the denominator into 10 or power of 10 and write $\frac{3}{4} = \frac{3 \times 25}{4 \times 25} = \frac{75}{100} = 0.75$

Example 2: Convert the following in the decimal form.

(i) $\frac{4}{5}$ (ii) $-\frac{2}{25}$ (iii) $\frac{3}{125}$ (iv) $\frac{7}{20}$

Solution: (i) $\frac{4}{5} = \frac{4 \times 2}{5 \times 2} = \frac{8}{10} = 0.8$

(ii) $\frac{-2}{25} = -\frac{2 \times 4}{25 \times 4} = -\frac{8}{100} = -0.08$

(iii) $\frac{3}{125} = \frac{3 \times 8}{125 \times 8} = \frac{24}{1000} = 0.024$

(iv) $\frac{7}{20} = \frac{7 \times 5}{20 \times 5} = \frac{35}{100} = 0.35$

Worksheet 1

1. Express the following rational numbers as decimals.

(i) $\frac{9}{8}$ (ii) $\frac{615}{125}$ (iii) $\frac{1}{16}$

(iv) $-\frac{3}{4}$ (v) $\frac{59}{200}$ (vi) $\frac{-24}{25}$

(vii) $\frac{-53}{250}$ (viii) $\frac{47}{400}$ (ix) $\frac{27}{800}$

(x) $\frac{139}{625}$ (xi) $\frac{3186}{1250}$ (xii) $\frac{133}{25}$

CONVERSION OF RATIONAL NUMBERS INTO DECIMALS BY LONG DIVISION METHOD

Every rational number can be represented in the form of decimal by using long division method. The representation can be either terminating or non-terminating but repeating.

I. Terminating Decimals

Consider the following examples.

Example 3: Express the following rational numbers as decimals by using long division method.

(i) $\frac{5}{8}$ (ii) $\frac{13}{5}$ (iii) $\frac{629}{125}$

Solution: To represent rational number in a decimal form, divide numerator by denominator.

(i) $$\begin{array}{r} 0.625 \\ 8\overline{)5.0} \\ \underline{48} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \\ \underline{0} \end{array}$$

(ii) $$\begin{array}{r} 2.6 \\ 5\overline{)13} \\ \underline{10} \\ 30 \\ \underline{30} \\ \underline{0} \end{array}$$

(iii) $$\begin{array}{r} 5.032 \\ 125\overline{)629} \\ \underline{625} \\ 400 \\ \underline{375} \\ 250 \\ \underline{250} \\ \underline{0} \end{array}$$

$\therefore \dfrac{5}{8} = 0.625$ $\qquad\therefore \dfrac{13}{5} = 2.6$ $\qquad\therefore \dfrac{629}{125} = 5.032$

Remember

Digits in the decimal part are called **decimal places.**

In the above examples when the numerator is divided by the denominator, we get the quotient with definite number of decimal places because the long division comes to an end after few steps.

Here, in each case, we get zero as the remainder and the quotient has a finite number of decimal places. Thus, the decimal obtained is called **Terminating Decimal**.

II. Non-Terminating Decimals

Let us consider the following examples.

Example 4: Convert the following rational numbers into decimal form.

(i) $\dfrac{1}{3}$ (ii) $\dfrac{2}{7}$ (iii) $\dfrac{25}{12}$

Solution: To represent $\dfrac{1}{3}$ as a decimal, we divide 1 by 3.

$$\begin{array}{r} 0.333... \\ 3\overline{)1.0} \\ \underline{9} \\ 10 \\ \underline{9} \\ 10 \\ \underline{9} \\ 1 \end{array}$$

Therefore, decimal form of $\dfrac{1}{3} = 0.333\ldots$

Here, the remainder 1 keeps on repeating again and again and 3 in the quotient also keeps on repeating.

When a digit goes on repeating endlessly, we place a bar (–) over it. Here, 3 is getting repeated, so we can write it as $\bar{3}$. Thus,

$$\frac{1}{3} = 0.333\ .\ .\ . = 0.\bar{3}$$

(ii) To represent $\frac{2}{7}$ as a decimal, we divide 2 by 7.

$$\begin{array}{r}
0.285714... \\
7\overline{)\,2.0} \\
\underline{14} \\
60 \\
\underline{56} \\
40 \\
\underline{35} \\
50 \\
\underline{49} \\
10 \\
\underline{7} \\
30 \\
\underline{28} \\
2
\end{array}$$

$$\therefore\ \frac{2}{7} = 0.285714\ .\ .\ . = 0.\overline{285714}$$

Here, we have remainder as 2 which is just the same as the dividend. Therefore, after 4, the same digits, i.e. 2, 8, 5, 7, 1, 4 will keep on repeating again and again in the quotient.

(iii) To represent $\frac{25}{12}$ as a decimal, we divide 25 by 12.

$$\begin{array}{r}
2.0833... \\
12\overline{)\,25.0} \\
\underline{24} \\
100 \\
\underline{96} \\
40 \\
\underline{36} \\
4
\end{array}$$

$\therefore\ \frac{25}{12} = 2.0833\ldots = 2.08\overline{3}$

Here, we have remainder as 4 which has appeared after the second step and then it is repeated. Therefore, the digit 3 in the quotient will keep on repeating.

From the above examples, we find that the division process does not terminate. Such numbers are called **Non-Terminating Repeating Decimals**.

So far we have converted positive rational numbers into decimal form. Now, we shall discuss how to convert negative rational numbers into decimal form.

Example 5: Find the decimal representation of the following rational numbers:

(i) $\frac{-5}{4}$ (ii) $\frac{-19}{7}$

Solution: (i) We know that $\frac{5}{4}$ in decimal form is represented as 1.25

Therefore, $\frac{-5}{4} = -1.25$

(ii) We also know that $\frac{19}{7}$ in decimal form is represented as $2.\overline{714285}$

Therefore, $\frac{-19}{7} = -2.\overline{714285}$

Now, let us see under which condition the decimal representation of a rational number terminates and under which condition it does not terminate. We take some examples to explain this.

Example 6: Find out whether the decimal representation of a rational number is terminating or non-terminating.

Solution:

$\frac{3}{2} = 1.5$ $\qquad \frac{3}{4} = 0.75$ $\qquad \frac{1}{8} = 0.125$

$\frac{2}{5} = 0.4$ $\qquad \frac{6}{25} = 0.24$ $\qquad \frac{2}{125} = 0.016$

$\frac{1}{10} = 0.1$ $\qquad \frac{1}{100} = 0.01$ $\qquad \frac{1}{1000} = 0.001$

To find out why we have terminating decimals in all the above examples, we observe prime factors of all the denominators.

$2 = 2$	$4 = 2 \times 2$	$8 = 2 \times 2 \times 2$
$5 = 5$	$25 = 5 \times 5$	$125 = 5 \times 5 \times 5$
$10 = 2 \times 5$	$100 = 2 \times 2 \times 5 \times 5$	$1000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5$

2 and 5 are the prime factors of all the denominators. Therefore, if we have only 2 and 5 as the prime factors of the denominator of a rational number in the lowest form, it will have terminating decimal representation. But, if the prime factors of the denominator are also other than 2 and 5, the decimal representation of that rational number (in the lowest form) will be a non-terminating repeating decimal.

Example 7: Without actual division, determine which of the following rational numbers have a terminating decimal representation?

(i) $\frac{21}{128}$ (ii) $\frac{27}{125}$ (iii) $\frac{39}{24}$ (iv) $-\frac{17}{90}$

Solution: We may note that all the above given numbers $\left(\text{except } \frac{39}{24}\right)$ are in the lowest form.

(i) The denominator of $\frac{21}{128}$ is 128.

$$128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

The prime factor of 128 is 2 seven times.

Therefore, $\frac{21}{128}$ has a terminating decimal representation.

(ii) The denominator of $\frac{27}{125}$ is 125.

$$125 = 5 \times 5 \times 5$$

The prime factor of 125 is 5 three times.

Therefore, $\frac{27}{125}$ has a terminating decimal representation.

(iii) The denominator of $\frac{39}{24}$ is 24.

$$24 = 2 \times 2 \times 2 \times 3$$

The prime factors of 24 are 2 and 3. One of the factors is other than 2 and 5. But the rational number is not in its lowest form.

In fact $\frac{39}{24} = \frac{13 \times 3}{8 \times 3} = \frac{13}{8}$, whose denominator $8 = 2 \times 2 \times 2$.

Therefore, $\frac{39}{24}$ has a terminating decimal representation.

(iv) The denominator of $\frac{17}{90}$ is 90.

$$90 = 2 \times 3 \times 3 \times 5.$$

The prime factors of 90 are 2, 3 and 5.

One of the factors is other than 2 and 5.

Therefore, $\frac{17}{90}$ will not have a terminating decimal representation.

Worksheet 2

1. Express the following rational numbers as decimals by using long division method.

(i) $\frac{21}{16}$ (ii) $\frac{129}{25}$ (iii) $\frac{17}{200}$ (iv) $\frac{5}{11}$

(v) $\frac{22}{7}$ (vi) $\frac{31}{27}$ (vii) $\frac{2}{15}$ (viii) $\frac{63}{55}$

2. Without actual division, determine which of the following rational numbers have a terminating decimal representation and which have a non-terminating decimal representation.

(i) $\frac{11}{4}$ (ii) $\frac{13}{80}$ (iii) $\frac{15}{11}$ (iv) $\frac{22}{7}$

(v) $\frac{29}{250}$ (vi) $\frac{37}{21}$ (vii) $\frac{49}{14}$ (viii) $\frac{126}{45}$

3. Find the decimal representation of the following rational numbers.

(i) $\frac{-27}{4}$ (ii) $\frac{-37}{60}$ (iii) $\frac{-18}{125}$ (iv) $\frac{-15}{8}$

4. If the number $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}$ is expressed as a decimal, will it be terminating or non-terminating? Justify your answer.

5. Justify the following statements as True or False.

(i) $\frac{22}{7}$ can be represented as a terminating decimal. ____________

(ii) $\frac{51}{512}$ can be represented as a terminating decimal. ____________

(iii) $\frac{19}{45}$ can be represented as a non-terminating repeating decimal. ______

(iv) $\frac{3}{17}$ cannot be represented as a non-terminating repeating decimal. ______

(v) If $\frac{3}{2}$ and $\frac{7}{5}$ are terminating decimals, then $\frac{3}{2} + \frac{7}{5}$ is also a terminating decimal. ______

(vi) If $\frac{1}{4}$ and $\frac{1}{5}$ both have terminating decimal representation, then $\frac{1}{4} \times \frac{1}{5}$ also has a terminating decimal representation. ______

CONVERSION OF TERMINATING DECIMALS INTO RATIONAL NUMBERS

In this section, we shall first discuss the process of converting a given decimal number into a rational number.

We take the following example to understand the process of conversion of a terminating decimal into the form $\frac{p}{q}$.

Example 7: Convert 1.2 in the form $\frac{p}{q}$.

Solution: Count the number of decimal places. In this case it is one.

Write the given number without the decimal point, i.e. 12.

Write a number with 12 in the numerator. The denominator is one followed by as many zeroes as is the number of decimal places in the given number.

In this case, the denominator will be 10.

Therefore, the required number is $\frac{12}{10}$.

Therefore, $1.2 = \frac{12}{10} = \frac{6}{5}$ (in the lowest form)

1 . 2

$$\frac{1\;2}{10} = \frac{12}{10}$$

Example 8: Express the following decimals in the form $\frac{p}{q}$.

(i) 0.4 (ii) 3.75 (iii) 1.025 (iv) 56.875 (v) – 2.56 (vi) – 0.25

Solution: (i) $0.4 = \frac{4}{10} = \frac{2}{5}$

(ii) $3.75 = \frac{375}{100} = \frac{15}{4}$

(iii) $1.025 = \frac{1025}{1000} = \frac{41}{40}$

(iv) $56.875 = \frac{56875}{1000} = \frac{455}{8}$

(v) $-2.56 = \frac{-256}{100} = \frac{-64}{25}$

(vi) $-0.25 = \frac{-25}{100} = \frac{-1}{4}$

Let us perform some operations on Decimal Numbers.

Example 9: Add 15.1, 12.03 and 7.209

Solution: First we convert these unlike decimal numbers into like decimal numbers then add as shown below.

$$\begin{array}{r} 15.100 \\ 12.030 \\ +\quad 7.209 \\ \hline 34.339 \\ \hline \end{array}$$

Example 10: Subtract 5.012 from 12.01

Solution: Convert into like decimal numbers and then subtract.

$$\begin{array}{r} 12.010 \\ -\quad 5.012 \\ \hline 6.998 \\ \hline \end{array}$$

Example 11: Multiply:

(i) 2.4 by 3.5 (ii) 4.8 by 1.84

Solution: (i) 2.4×3.5

First multiply 24 and 35 without decimal point.

$$\begin{array}{r} 24 \\ \times\quad 35 \\ \hline 120 \\ +\quad 720 \\ \hline 840 \\ \hline \end{array}$$

$24 \times 35 = 840$

$2.4 \times 3.5 = 8.40 = 8.4$

We find that the sum of decimal places in the given numbers is 1 + 1 = 2. So, the required product is 8.40 or 8.4.

(ii) 4.8 by 1.84

$$\begin{array}{r} 184 \\ \times \quad 48 \\ \hline 1472 \\ + \; 7360 \\ \hline 8832 \\ \hline \end{array}$$

$48 \times 184 = 8832$

$4.8 \times 1.84 = 8.832$

We find that the sum of the number of decimal places in the given two numbers is 1 + 2 = 3. So, the required product is 8.832.

Example 12: Divide:

(i) 32.768 by 8 (ii) 6.25 by 0.5.

Solution: (i) $32.768 \div 8$

$$\begin{array}{r} 4.096 \\ 8\overline{)32.768} \\ \underline{32} \\ 76 \\ \underline{72} \\ 48 \\ \underline{48} \\ 0 \\ \hline \end{array}$$

Therefore, $32.768 \div 8 = 4.096$

(ii) $6.25 \div 0.5$

Now, $\frac{6.25}{0.5} = \frac{625}{10\cancel{0}} \times \frac{1\cancel{0}}{5}$

$= \frac{625}{50} = \frac{125}{10} = 12.5$

Example 13: Simplify and express the result in the decimal form.

$\frac{1}{5} + \frac{3}{10} + \frac{4}{25}$

Solution: $\frac{1}{5}+\frac{3}{10}+\frac{4}{25} = \frac{10+15+8}{50}$

$= \frac{33}{50} = 0.66$

$$\begin{array}{r} 0.66 \\ 50\overline{)33.0} \\ \underline{300} \\ 300 \\ \underline{300} \\ \underline{0} \end{array}$$

Worksheet 3

1. Express the following decimals as rational numbers in standard form.

(i) 0.25 (ii) – 0.052 (iii) 7.50

(iv) – 2.15 (v) 0.036 (vi) – 9.6

(vii) 31.25 (viii) 16.32 (ix) 0.107

2. Add the following decimals.

(i) 2.5, 7.51 and 11.501 (ii) 12, 5.96 and 3.076 (iii) 9.08, 19.76 and 20.54

(iv) 3.009, 0.592 and 14.745 (v) 19, 9.5, 12.06 and 17.921

3. Compute the following products of decimals.

(i) 2.9×3.5 (ii) 37×12.76 (iii) 0.84×8.8

(iv) 2.56×11.09 (v) $12.4 \times 15.7 \times 13.2$

4. Compute the following divisions.

(i) $59.049 \div 9$ (ii) $6.4 \div 0.2$ (iii) $0.015 \div 3$

(iv) $0.014 \div 12$ (v) $0.02472 \div 0.008$ (vi) $51.51 \div 0.17$

5. Evaluate the following:

(i) $25.75 + 2.09 – 13.6$ (ii) $37 – 16.58 + 12.25$

(iii) $42.7 – 11 – 9.025 + 2.16$ (iv) $(6.05 + 5.01) – (12.5 – 0.09)$

(v) $182.3 + 12.65 – 0.23 – 10.71$

6. Simplify and express the result as a rational number in its lowest terms.

(i) $\frac{1}{2}+\frac{1}{5}+6.25 \div 0.25$ (ii) $\frac{2}{5}-\frac{1}{4}+(8.1\times 2.7)\div 0.09$

(iii) $1.44 \times (144 \div 12) – 0.225 + 3.276$ (iv) $\frac{1}{7}\times 0.049+\frac{3}{8}-\frac{7}{20}$

(v) $5 \times 0.16 – 0.52 + 8.263$ (vi) $\frac{2}{5}\times\frac{3}{4}+\frac{1}{25}\times\frac{1}{2}-\frac{2}{10}\times\frac{1}{5}$

VALUE BASED QUESTIONS

1. Megha bought a book for ₹ $112\frac{1}{2}$ from a shop. She gave 500 rupee note to the shopkeeper and got the balance back. But, she realised that the shopkeeper had given her ₹ 72 extra. Megha returned the extra money and had a feeling of great satisfaction.

(a) How much money had the shopkeeper returned to Megha?

(b) What values did Megha exhibit in the above situation?

2. Raman had to cover a distance of 30 km to reach his grandmother's house. He covered 11.25 km by bus, 7.083 km by auto and rest by foot.

(a) How much distance did Raman cover by foot?

(b) How is Raman benefitted if he walks down to any of his destination? In what ways does it effect out environment?

BRAIN TEASERS

1. A. Tick (✓) the correct option.

(a) 0.225 expressed as a rational number is–

(i) $\frac{1}{4}$ (ii) $\frac{45}{210}$ (iii) $\frac{9}{40}$ (iv) $\frac{225}{999}$

(b) A rational number $\frac{p}{q}$ can be expressed as a terminating decimal if q has no prime factor other than–

(i) 2, 3 (ii) 2, 5 (iii) 3, 5 (iv) 2, 3, 5

(c) $-7\frac{8}{100}$ expressed as a decimal number is–

(i) – 7.800 (ii) 7.008 (iii) – 7.008 (iv) – 7.08

(d) $4.01\overline{325}$ is equal to–

(i) 4.013252525... (ii) 4.0132555...

(iii) 4.0132501325... (iv) 4.0130132525...

(e) The quotient when 0.00639 is divided by 0.213 is–

(i) 3 (ii) 0.3 (iii) 0.03 (iv) 0.003

B. Answer the following questions.

(a) Without actual division, determine if $\frac{-28}{250}$ is terminating or non-terminating decimal number.

(b) Convert $\frac{-113}{7}$ to decimals.

(c) What should be subtracted from – 15.834 to get 3.476?

(d) Express 4.82 as rational number in standard form.

(e) Find the value of 16.016 ÷ 0.4.

2. Convert the following rational numbers into decimals.

(i) $\frac{259}{3}$ (ii) $\frac{19256}{11}$ (iii) $\frac{15735}{80}$

(iv) $\frac{27}{7}$ (v) $\frac{758}{1250}$ (vi) $\frac{15625}{12}$

3. Find the decimal representation of the following rational numbers.

(i) $-\frac{12}{13}$ (ii) $-\frac{1525}{50}$ (iii) $-\frac{127}{7}$ (iv) $-\frac{539}{80}$

4. Simplify the following expressions.

(i) 3.2 + 16.09 + 26.305 – 1.232 (ii) – 5.7 + 13.20 – 15.009 + 0.02

(iii) (0.357 + 0.96) – (3.25 – 2.79) (iv) 15 + 2.57 – 23.07 – 5.003

5. Without actual division, determine which of the following rational numbers have a terminating decimal representation.

(i) $\frac{327}{125}$ (ii) $\frac{99}{800}$ (iii) $\frac{17}{1250}$ (iv) $\frac{29}{200}$

(v) $\frac{135}{1625}$ (vi) $\frac{1276}{680}$ (vii) $\frac{22}{190}$ (viii) $\frac{11}{750}$

6. Simplify the following and express the result as decimals.

(i) 2.7 × 1.5 × 2.1 (ii) 12 × 13.6 × 0.25

(iii) 3.25 × 72.6 (iv) (156.25 ÷ 0.025) × 0.02 – 5.2

(v) (75.05 ÷ 0.05) × 0.001 + 2.351

7. Simplify and express the result as a rational number in its lowest form.

(i) $3.125 \div 0.125 + 0.50 - 0.225$

(ii) $\dfrac{0.4 \times 0.04 \times 0.005}{0.1 \times 10 \times 0.001} - \dfrac{1}{2} + \dfrac{1}{5}$

(iii) $\dfrac{0.144 \div 1.2}{0.016 \div 0.02} + \dfrac{7}{5} - \dfrac{21}{8}$.

HOTS

Perimeter of a rectangle is 2.4 m less than $\frac{2}{5}$ of the perimeter of a square.

If the perimeter of the square is 40 m, find the length and breadth of the rectangle given that breadth is $\frac{1}{3}$ of the length.

ENRICHMENT QUESTION

There is an interesting pattern in the following:

$$\frac{1}{7} = 0.\overline{142857}$$
$$\frac{2}{7} = 0.\overline{285714}$$
$$\frac{3}{7} = 0.\overline{428571}$$
$$\frac{4}{7} = 0.\overline{571428}$$
$$\frac{5}{7} = 0.\overline{714285}$$
$$\frac{6}{7} = 0.\overline{857142}$$

$$\frac{3}{7} + \frac{4}{7} = 0.\overline{9} \rightarrow \frac{2}{7} + \frac{5}{7} = 0.\overline{9} \rightarrow \frac{1}{7} + \frac{6}{7} = 0.\overline{9}$$

You will question why the left hand side in each case is 1, but, the right hand side is $0.\overline{9}$? (You will learn about this in higher classes).

Notice they all have only the digits 142857, each starting with a different digit but in the same order.

Try finding out the repeating part of the decimal for $\frac{1}{13}$. What do you notice?

YOU MUST KNOW

1. Every rational number can be represented as a decimal.
2. The decimal representation of a rational number is either terminating or non-terminating but repeating.
3. Decimal numbers having a finite number of decimal places are known as terminating decimal numbers.
4. Decimal numbers having an infinite number of decimal places are known as non-terminating decimal numbers.
5. Decimal numbers having an infinite number of decimal places and a set of digits in the decimal places that repeat are known as non-terminating repeating decimal numbers.
6. If the denominator of a rational number in the standard form has 2 or 5 or both as the only prime factors, then it can be represented as a terminating decimal.
7. If the denominator of a rational number in the standard form has prime factors other than 2 and 5, then it cannot be represented as a terminating decimal. In fact, it is a non-terminating but repeating decimal.