SETS

Def^n A well defined collection of objects is called Set.

It is represented by Capital Alphabet

for example i) $A = \{2, 4, 6, 8, 10\}$ (Set of first five even numbers)

Here $n(A) = 5$

ii) Let B → Set of first 3 prime number

$B = \{2, 3, 5\}$ Here $n(B) = 3$

Number of elements in B is three

In above set B [redacted] $2 \in B$ (belongs to) (we read 2 belongs to B).

* $n(C)$ → number of elements in set C

Representation of Sets
1) Roster form
2) Set Builder form

i) Roster form (Tabulation Method)

In this Method, we list all the members (elements) of set within { } (middle bracket) and separate them by commas

$N = \{1, 2, 3, 4, 5\}$

Q. Write the set J which contains all months having 31 days

Ans $J = \{$January, March, May, July, August, October, December$\}$

Here $n(J) = 7$

Q. Write in roster form E → set of all letters in the word 'TRIGONOMETRY'

Ans $E = \{T, R, I, G, O, N, M, E, Y\}$

* In set notation repetition of element has no meaning.

2) <u>Set Builder form</u> :- In this method, we list the property or properties satisfied by all the elements of the set.

or write in generalize way.

<u>for example</u> If $A = \{1, 2, 3, 4, 5\}$ (given)

Q Write A in Set Builder form

<u>Ans</u> $A = \{x : x \in N, x \leq 5\}$

element name — x; such that — $:$; belongs to — $\in$; natural numbers — N

Q. $A = \{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}\}$; write A in Set Builder form

<u>Ans</u> $A = \{x : x = \frac{n}{n+1}; n < 5; n \in N\}$

Q $B = \{2, 4, 8, 16, 32, ----\}$

Write B in Set Builder form

<u>Ans</u> $B = \{x : x = 2^n; n \in N\}$

* Here B is infinite set

Practice Questions (1)

1) Write in Roster form :-

a) $A = \{x : x$ is natural number, $30 \leq x < 36\}$

b) $B = \{x : x$ is perfect square and $x < 50\}$

c) $C = \{x : x$ is a two digit number such that sum of its digits is $9\}$

d) $D = \{x : x = n^2, n \in N$ and $2 \leq n \leq 5\}$

Cont.

Practice (2) Write each of the sets in Set-Builder form

a) $H = \{14, 21, 28, 35, 42, ---, 98\}$

b) $K = \{1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \frac{1}{36}, \frac{1}{49}\}$

c) $J = \{3, 9, 27, 81, 243\}$

Remark

$N \rightarrow$ natural number

$Z \rightarrow$ Set of Integers. or I

$Q \rightarrow$ Set of Rational numbers.

$R \rightarrow$ Set of Real numbers.

Practice (3) Match the following :-

Roster form	Set Builder form
(i) $\{-1, 1\}$	a) $\{x : x \in Z \text{ and } x^2 < 16\}$
ii) $\{1, 2, 3, 6, 9, 18\}$	b) $\{x : x \in N \text{ and } x^2 = x\}$
iii) $\{-3, -2, -1, 0, 1, 2, 3\}$	c) $\{x : x \in Z \text{ and } x^2 = 1\}$
iv) $\{P, R, I, N, C, A, L\}$	d) $\{x : x \in N$ and x is a factor of $18\}$
v) $\{1\}$	e) $\{x : x$ is letter in the word 'PRINCIPAL'$\}$

Types of Sets

i) Empty set / void set / Null set

A set containing no element is called null set.
It is denoted by ϕ or $\{\}$

↓
phi

for example: Write set A which contains even prime other than 2

~~Ans~~:- $A = \{\}$ Here $n(A) = 0$

ii) Singleton set

A set containing exactly one element.

for example Write set B which contains integer ~~with~~ which is neither +ve nor -ve

$B = \{0\}$ Here $n(B) = 1$

iii) Finite set

Set contains countable number of elements.

for example $A = \{1, 2, 3, 4, 7, 8\}$
Here $n(A) = 6$ (finite)

iv) Infinite set

Set contains uncountable number of elements

for example $N = \{1, 2, 3, 4, 5, ----\}$
natural nos

$Z = \{---, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4 ----\}$
Integers

$W \rightarrow$ whole number $= \{0, 1, 2, 3, ---\}$

v) **Equal Sets** Two nonempty sets A and B are said to be equal, if they have exactly the same elements and ~~and~~ we write $A = B$

for example $A = \{1, 2, 3\}$, $B = \{x : x \in N, x \leq 3\}$

Here $A = B$

Q Let A = Set of letters in the word 'follow'
Practice
B = Set of letters in the word 'wolf'

Show that $A = B$

vi) **Equivalent sets** Two finite sets A and B are said to be equivalent if $n(A) = n(B)$

for example $A = \{1, 2, 3\}$, $B = \{x, y, z\}$

Here $n(A) = 3$

$n(B) = 3$

$\Rightarrow A \sim B$ (A is equivalent of B)
But $A \neq B$ (as elements are not same)

Remark Equal sets are equivalent also.

Q Show that $\{0\}$ and ϕ are not equivalent sets.
phi

Sol Let $A = \{0\}$ Here $n(A) = 1$ since $1 \neq 0 \Rightarrow A \not\sim B$
$B = \phi$ Here $n(B) = 0$ (not equivalent)

Practice Let $A = \{x : x \in N, x^2 - 9 = 0\}$
$B = \{x : x \in Z, x^2 - 9 = 0\}$

Show that $A \neq B$

Practice Questions (upto Infinite sets)

Q1) Which of the following are examples of the singleton set?

a) $A = \{x : x \in Z, x^2 = 4\}$

b) $B = \{x : x \in Z, x + 5 = 0\}$

c) $C = \{x : x \in N, x^2 = 16\}$

Q2. Which of the following are pairs of equal sets?

a) A = set of letters in the word, 'ALLOY'
B = set of letters in the word, 'LOYAL'

b) $C = \{2, 3\}$
$D = \{x : x \in Z, x^2 + 5x + 6 = 0\}$

c) $E = \{a, e, i, o, u\}$
$F = \{p, q, r, s, t\}$

Q3 Which of the following sets are finite or Infinite?

a) Set of all lines parallel to the y-axis.

b) $H = \{x : x \in Z; -15 < x < 15\}$

c) $K = \{x : x \in R; 0 < x < 1\}$

d) $B = \{x : x \in N$ and x is prime$\}$

Q4 Rewrite the following statements using set notation:

a) Number of elements in A is 5

b) a is an element of A

c) 0 is a whole number but not a natural number

Hint $0 \in W$ but $0 \notin N$

SUBSETS

Subset A set A is said to be subset of set B If every element of A is also an element of B, and denoted by $A \subseteq B$ (A is subset of B)

Here B → Superset of A

If $A = \{1, 2, 3\}$ then number of subsets $= 2^3 = 8$

* If a set contains n elements then number of subsets are 2^n.

Proper Subset $A \subset B$ (A is Proper Subset of B)

For example $A = \{1, 2, 3\}$

Subsets are $\{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\}, \phi$

Proper Subsets are $\{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \phi$

Formula
(2^n)
$(2^n - 1)$

Remark 1) $N \subset W \subset Z \subset Q \subset R$

N → natural number, W → whole number, Z → Integers, Q → Rational, R → Real

2) Every set is a subset of itself.

3) Empty set is a subset of every set.

4) No of Proper Subsets of set A < No of Subsets of set A

their difference = 1

* Universal Set U

- Superset of all given sets.
- All the sets are subsets of U

for example For even natural numbers / odd numbers, set of natural numbers N be considered as Universal set.

Q. If $A = \{1, 3, 5\}$, $B = \{2, 4, 6\}$, $C = \{0, 2, 4, 6, 8\}$, then find U.

* Power Set

It is a set of all the subsets of a given set.

for example $A \rightarrow$ given set (n elements)

$P(A) \rightarrow$ Power set of A (2^n elements)

Q $A = \{1, 2\}$, write $P(A)$

Ans $P(A) = \{\{1\}, \{2\}, \{1, 2\}, \phi\}$. Here $n(P(A)) = 4$

* Intervals (as subsets of R).

Let $a, b \in R$ (Real numbers), $a < b$

1) Closed Interval $\rightarrow$ $[a, b] = \{x : x \in R; a \leq x \leq b\}$

ii) Open Interval $\rightarrow$ $(a, b) = \{x : x \in R; a < x < b\}$
or $]a, b[$

iii) Semi Closed Semi Open $\rightarrow$ $[a, b) = \{x : x \in R; a \leq x < b\}$

iv) Semi open semi closed $\rightarrow$ $(a, b] = \{x : x \in R; a < x \leq b\}$

Q. Write the following sets in interval form :-

$A = \{x : x \in R ; -5 < x < -1\}$

Sol $A = (-5, -1)$

Q. Write $B = [-4, 7]$ in set Builder form.

$B = \{x : x \in R ; -4 \leq x \leq 7\}$

Practice Questions (1) Write each of following sets in Interval form

i) $A = \{x : x \in R ; -4 \leq x < 0\}$
ii) $B = \{x : x \in R ; -2 \leq x \leq 2\}$
iii) $C = \{x : x \in R ; 7 < x \leq 10\}$

(2) Write each of the following intervals in set builder form:

i) $A = (-1, 3)$
ii) $B = [2, 4]$
iii) $C = [6, 12[$

(3) Examine whether the following statements are T or F

i) $\{a, b\} \not\subset \{b, c, a\}$
ii) $\{a, e\} \subset \{x : x$ is vowel in the English alphabet$\}$
iii) $\{b, c\} \subset \{a, \{b, c\}\}$ (Here $\{b, c\}$ is one of the element)
iv) If A = set of all circles of unit radius in a plane and B = set of all circles in same plane then $A \subset B$

OPERATIONS ON SETS

Union of Sets The union of two sets A and B, denoted by $A \cup B$, is the set of all those elements which are either in A or in B or in both.

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

If $A = \{1, 2, 3\}$ $B = \{2, 3, 4, 5\}$

then $A \cup B = \{1, 2, 3, 4, 5\}$

Intersection of Sets The intersection of two sets A and B is denoted by $A \cap B$, is the set of all elements which are common to both A and B.

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

If $A = \{1, 2, 3\}$, $B = \{2, 3, 4, 5\}$ then $A \cap B = \{2, 3\}$

Q. Let $A = \{x : x \text{ is a positive integer}\}$

$B = \{x : x \text{ is a negative integer}\}$

Here $A \cup B = \{x : x \text{ is an integer and } x \neq 0\}$

$A \cap B = \phi$

Disjoint sets Two sets A and B are said to be disjoint if $A \cap B = \phi$ (nothing common)

Difference of Sets

for two set A and B

$A - B = \{x : x \in A \text{ and } x \notin B\}$

$B - A = \{x : x \in B \text{ and } x \notin A\}$

for example $A = \{1, 2, 3\}$ $B = \{3, 4, 5\}$

$A - B$ = Set of those element of A which are not in B

$= \{1, 2\}$

$B - A = \{4, 5\}$

Practice Question If $A = \{x : x \in N, x \text{ is factor of } 6\}$

$B = \{x : x \in N, x \text{ is factor of } 8\}$

find i) $A \cup B$ ii) $A \cap B$ iii) $A - B$ iv) $B - A$

Hint Here $A = \{1, 2, 3, 6\}$ and $B = \{1, 2, 4, 8\}$

Complement of Set

Let U be the universal set and let $A \subset U$ then complement of $A = A'$ or A^c

$A' = U - A$

For example $U = \{2, 3, 5, 7, 8, 10\}$ $A = \{3, 5\}$

$A' = \{2, 7, 8, 10\}$

Practice Let N be universal set and $A = \{x : x \in N, x \text{ odd}$

Write A'

Practice $A = \{2, 4, 6, 8, 10, 12\}$

$B = \{3, 4, 5, 6, 7, 8, 10\}$

find i) $A - B$ ii) $B - A$ iii) $(A-B) \cup (B-A)$

iv) $A \cup B$ v) $A \cap B$

Fill in the Blanks ($U \to$ universal set)

i) $A \cup A' =$ ______

ii) $A \cap A' =$ ______

iii) $\phi' \cap A =$ ______

iv) $U' \cap A =$ ______

v) $(A')' =$ ______

Law of operations on Sets

i) Commutative Law a) $A \cup B = B \cup A$

b) $A \cap B = B \cap A$

ii) Associative Law a) $(A \cup B) \cup C = A \cup (B \cup C)$

b) $(A \cap B) \cap C = A \cap (B \cap C)$

iii) Distributive Law a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Imp iv) Demorgan's Law a) $(A \cup B)' = A' \cap B'$

b) $(A \cap B)' = A' \cup B'$

Practice Q. If $A = \{x : x \in N, x \leq 7\}$

$B = \{x : x \text{ is prime}, x < 8\}$

$C = \{x : x \in N, x \text{ is odd and } x < 10\}$

Verify that

i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

~~[illegible]~~

iii) $A - (B \cap C) = (A - B) \cup (A - C)$

Practice Ques If $U = \{a, b, c, d, e, f\}$

$A = \{a, b, c\}$,

$B = \{b, c, d, e\}$

$C = \{c, d, e, f\}$

Verify that i) $(A')' = A$

ii) $(A \cup B)' = A' \cap B'$

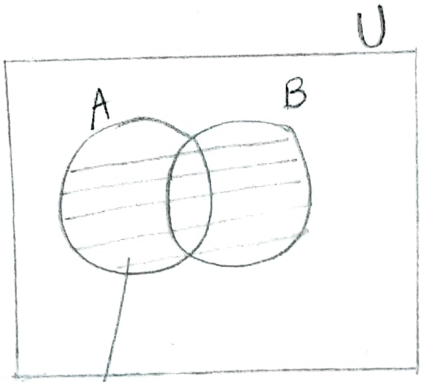
iii) $(A \cap B)' = A' \cup B'$

(iv) $A \cap (B - C) = (A \cap B) - (A \cap C)$

Note For calculating complement of a set, Universal set must be given.

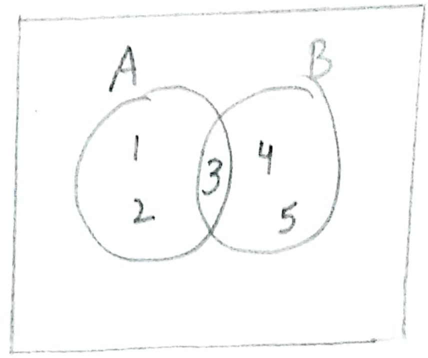
Venn Diagrams.

(1)



$A \cup B$ (Shaded Part).

Example

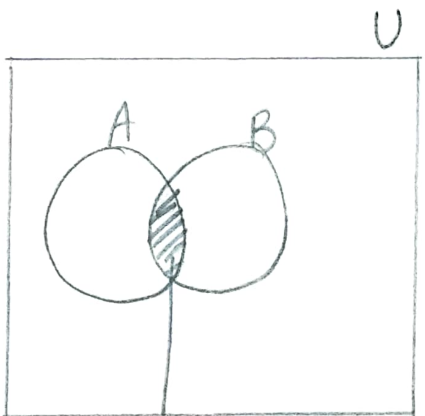


$A = \{1,2,3\}$ $B = \{3,4,5\}$

$A \cup B = \{1,2,3,4,5\}$

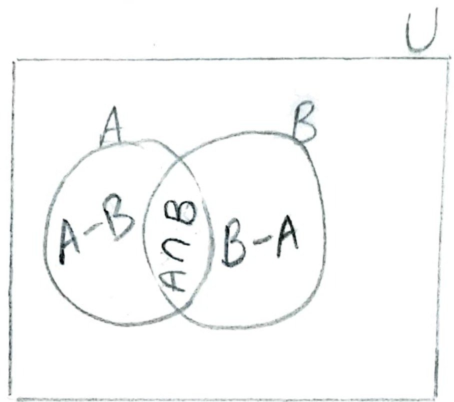
$A \cap B = \{3\}$

(2)



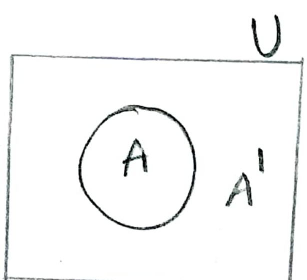
$A \cap B$

(3)



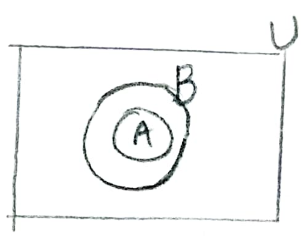
* $A-B$, $A \cap B$, $B-A$ are disjoint sets.

(4)

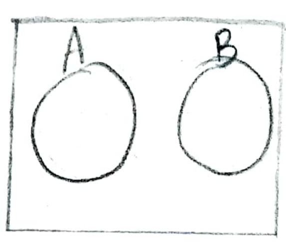


$A' = U - A$

(5) If $A \subset B \subset U$, then



(6)



Here A and B are disjoint sets.

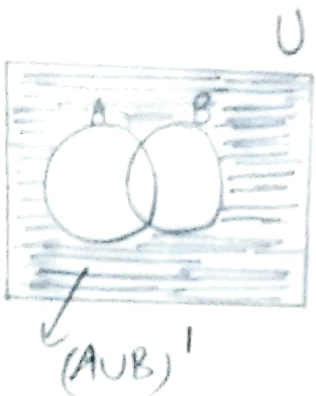
Q Using Venn diag, verify Demorgan's law

Sol We know Demorgan's law

i) $(A \cup B)' = A' \cap B'$

ii) $(A \cap B)' = A' \cup B'$

(i) LHS

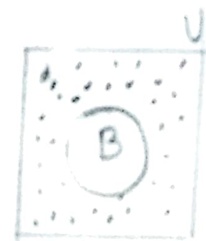


RHS

fig 1

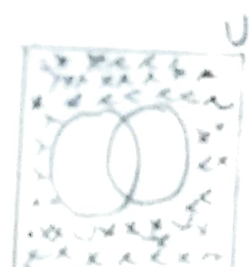
$= A'$

fig 2



$= B'$

fig 3



$= A' \cap B' =$ LHS

(ii) Practice Yourself

Q(Practice) Using venn diagram, Show that $(A-B), (B-A), (A \cap B)$ are disjoint sets,

where $A = \{2, 4, 6, 8, 10, 12\}$

$B = \{3, 6, 9, 12, 15\}$

Results derived from Venn diag

i) $n(A\cup B) = n(A-B) + n(B-A) + n(A\cap B)$

*ii) $n(A\cup B) = n(A) + n(B) - n(A\cap B)$

iii) $n(A) = n(A-B) + n(A\cap B)$

iv) $n(B) = n(B-A) + n(A\cap B)$

*v) for three sets A, B, C

$$n(A\cup B\cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(A\cap C) + n(A\cap B\cap C)$$

vi) If A and B are disjoint sets, then

$n(A\cup B) = n(A) + n(B)$

Q In a survey of 420 students in a school, it was found that 120 drink apple juice, 150 drink orange juice and 80 drink both. How many drink neither apple juice nor orange juice?

Sol Let U → Set of all students in school

A → Students who drink apple juice

B → Students who drink orange juice

Here $n(U) = 420$, $n(A) = 120$, $n(B) = 150$, $n(A\cap B) = 80$

To find $n(A'\cap B') = ?$

Sol We know $A'\cap B' = (A\cup B)'$ (by demorgan's law)

$\Rightarrow n(A\cup B)' = n(U) - n(A\cup B) = n(U) - \{n(A) + n(B) - n(A\cap B)\}$

$= 420 - (120 + 150 - 80) = 230$

Practice Ques ① In a group of 52 persons, 16 drink tea but not coffee and 33 drink tea. Find

a) how many drink tea and coffee both?

b) how many drink coffee but not tea?

② In a class of a certain school, 50 students offered mathematics, 42 offered biology and 24 offered both the subjects. Find the number of students offering

a) mathematics only

b) biology only

c) any of the two subjects.

Q. 3 Sets. In a survey of 25 students, it was found that 12 have taken physics, 11 have taken chemistry and 15 have taken mathematics, 4 have taken Phy. and chem., 9 have taken Phy. and maths, 5 have taken chem. and maths ~~which~~ while 3 have taken all the three subjects.

Draw Venn diag, Hence find number of students.

i) Physics only

ii) Chem. only

iii) maths only

iv) exact one subject

v) exact two subjects

vi) Physics and chem. only

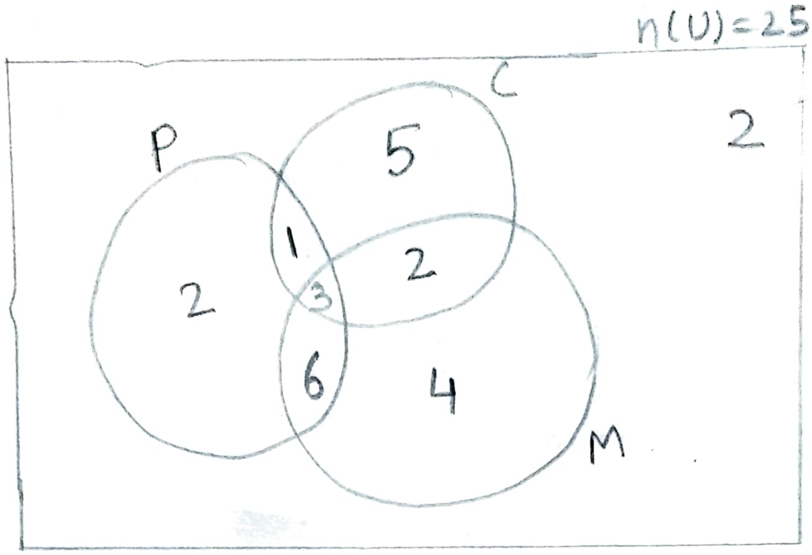
vii) atleast one of three subjects

viii) none of three subjects

ix) Phy. & Chem. but not maths

x) atleast two subjects.

Venn Diag.



P → Phy   C → Chem.   M → Maths

i) n(P only) = 2

ii) n(C only) = 5

iii) n(M only) = 4

iv) n(exact one subject) = 2+5+4 = 11

v) n(exact two subjects) = 1+6+2 = 9

vi) n(P and C only) = 1

vii) n(atleast one of 3 subjects) = 2+1+5+6+3+2+4
or n(PUMUC) = 23

viii) n(none of 3 subjects) = n(U) − n(PUMUC)
= 25 − 23
= 2

ix) n(P or C but not M) = 2+1+5 = 8

x) n(atleast 2 subjects) = n(2 subjects) + n(3 subjects)
= 1+2+3+6 = 12.

Practice Questions

(1) In a survey of 100 students, the number of students studying the various languages is found as : English only 18, English but not Hindi 23; English and Sanskrit 8; Sanskrit and Hindi 8, English 26, Sanskrit 48 and no language 24. ~~find~~

i) Draw Venn diag

ii) find, how many students are studying Hindi ?

iii) find, how many students are studying English and Hindi both ?

iv) find, how many students are studying Sanskrit only ?

v) find, how many students are studying atleast one of three languages ?

* Do Misc. EX (Last Questions).

1) For any two sets A and B, Prove that

a) $A \cup B = B \cup A$ b) $A \cap B = B \cap A$

Pf:- a) Case-1 Let x be an arbitrary element of $A \cup B$

$\Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in B$ or $x \in A$

$\Rightarrow x \in B \cup A$

$\Rightarrow A \cup B \subseteq B \cup A$ —— (1)

Case-2 Let y be an arbitrary element of $B \cup A$

$\Rightarrow y \in B$ or $y \in A$

$\Rightarrow y \in A$ or $y \in B$

$\Rightarrow y \in A \cup B$

$\Rightarrow B \cup A \subseteq A \cup B$ —— (2)

from (1) & (2) $A \cup B = B \cup A$, Hence Proved

Pf b) Let x be an arbitrary element of $A \cap B$

Case-1 $\Rightarrow x \in A$ and $x \in B$

$\Rightarrow x \in B$ and $x \in A$

$\Rightarrow x \in B \cap A$

$\Rightarrow A \cap B \subseteq B \cap A$ —— (1)

Case-2 Let y be an arbitrary element of $B \cap A$

$\Rightarrow y \in B$ and $y \in A$

$\Rightarrow y \in A$ and $y \in B$

$\Rightarrow y \in A \cap B$

$\Rightarrow B \cap A \subseteq A \cap B$ —— (2)

from (1) & (2) $A \cap B = B \cap A$, Hence Proved

(2) For any sets A, B, C, Prove that

a) $(A \cup B) \cup C = A \cup (B \cup C)$

b) $(A \cap B) \cap C = A \cap (B \cap C)$

Sol a) For Proving $(A \cup B) \cup C = A \cup (B \cup C)$

we have to Prove $(A \cup B) \cup C \subseteq A \cup (B \cup C)$

and $A \cup (B \cup C) \subseteq (A \cup B) \cup C$

Case-1 Let $x \in (A \cup B) \cup C$ ($x \to$ ~~arbitrary~~ any element)

$\Rightarrow x \in (A \cup B)$ or $x \in C$

$\Rightarrow (x \in A$ or $x \in B)$ or $x \in C$

$\Rightarrow x \in A$ or $(x \in B$ or $x \in C)$

$\Rightarrow x \in A$ or $(x \in B \cup C)$

$\Rightarrow x \in A \cup (B \cup C) \Rightarrow (A \cup B) \cup C \subseteq A \cup (B \cup C)$ — (1)

Case-2 Let y be arbitrary element of $A \cup (B \cup C)$

$\Rightarrow y \in A$ or $y \in (B \cup C)$

$\Rightarrow y \in A$ or $(y \in B$ or $y \in C)$

$\Rightarrow (y \in A$ or $y \in B)$ or $y \in C$

$\Rightarrow y \in (A \cup B)$ & $y \in C$

$\Rightarrow y \in (A \cup B) \cup C$

$\Rightarrow A \cup (B \cup C) \subseteq (A \cup B) \cup C$ — (2)

from (1) and (2) $(A \cup B) \cup C = A \cup (B \cup C)$

b) Practice Yourself

Hint If $x \in (A \cap B) \cap C$

then $x \in (A \cap B)$ and $x \in C$

$\Rightarrow x \in A$ and $x \in B$ and $x \in C$ Now Rearrange acc.

3 Distributive Property for three sets A, B, C 3

To Prove a) $A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$

b) $A\cap(B\cup C) = (A\cap B)\cup(A\cap C)$

Pf:- a) Case-1 Let x be any element of $A\cup(B\cap C)$

$\Rightarrow x\in A$ or $x\in(B\cap C)$

$\Rightarrow x\in A$ or $(x\in B$ and $x\in C)$

$\Rightarrow (x\in A$ or $x\in B)$ and $(x\in A$ or $x\in C)$

$\Rightarrow x\in(A\cup B)$ and $x\in(A\cup C)$

$\Rightarrow x\in(A\cup B)\cap(A\cup C)$

$\Rightarrow A\cup(B\cap C) \subseteq (A\cup B)\cap(A\cup C)$ (1)

Case-2 Let y be any element of $(A\cup B)\cap(A\cup C)$

$\Rightarrow y\in(A\cup B)$ and $y\in(A\cup C)$

$\Rightarrow (y\in A$ or $y\in B)$ and $(y\in A$ or $y\in C)$

$\Rightarrow y\in A$ or $(y\in B$ and $y\in C)$

$\Rightarrow y\in A$ or $(y\in B\cap C)$

$\Rightarrow y\in A\cup(B\cap C)$ $\Rightarrow (A\cup B)\cap(A\cup C) \subseteq A\cup(B\cap C)$ — (2)

from (1) & (2) $(A\cup B)\cap(A\cup C) = A\cup(B\cap C)$

b) Practice Yourself

(4) Verify Demorgan's Law (4)

(1) $(A \cup B)' = A' \cap B'$ (2) $(A \cap B)' = A' \cup B'$

Pf: (1) Case-1 Let $x \in (A \cup B)'$

$\Rightarrow x \notin (A \cup B)$

$\Rightarrow x \notin A$ and $x \notin B$ [Note in -ve sense or $\rightarrow$ and]

~~$\Rightarrow x \notin A$~~

$\Rightarrow x \in A'$ and $x \in B'$

$\Rightarrow x \in A' \cap B'$

$\Rightarrow (A \cup B)' \subseteq A' \cap B'$ —— (*)

Case-2 Let $y \in A' \cap B'$

$\Rightarrow y \in A'$ and $y \in B'$

$\Rightarrow y \notin A$ or $y \notin B$

$\Rightarrow y \notin (A \cup B)$

$\Rightarrow y \in (A \cup B)'$ $\Rightarrow A' \cap B' \subseteq (A \cup B)'$ —— (**)

from (*) and (**) $(A \cup B)' = A' \cap B'$

(2) Practice Yourself $\cap B']$

(5) For any two sets A and B, To Prove if $A \subseteq B$ then $B' \subseteq A'$

Proof Let $x \in B'$

$\Rightarrow x \notin B$

$\Rightarrow x \notin A$ (as B is superset of A given) [tive ty]

$\Rightarrow x \in A'$

Thus $B' \subseteq A'$, Hence Proved

(5)

(6) Given $A \cup B = A \cap B$

To Prove $A = B$

Proof:- Case-1 To Prove $A \subseteq B$

Let $x \in A$

$\Rightarrow x \in A \cup B$

$\Rightarrow x \in A \cap B$ (given)

$\Rightarrow x \in A$ and $x \in B$

$\Rightarrow x \in B$

$\Rightarrow A \subseteq B$ — (1)

Case-2 To Prove $B \subseteq A$

Let $y \in B$

$\Rightarrow y \in A \cup B$

$\Rightarrow y \in A \cap B$ (given)

$\Rightarrow y \in A$ and $y \in B$

$\Rightarrow y \in A$

$\Rightarrow B \subseteq A$ — (2)

from (1) & (2)

$A = B$

---

(7) for any sets A and B, Prove that

i) $(A \cap B) \cup (A - B) = A$

ii) $A \cup (B - A) = A \cup B$

Proof (i) Take LHS $(A \cap B) \cup (A - B)$

$= (A \cap B) \cup (A \cap B')$ [$\because A - B = A \cap B'$]

$= A \cap (B \cup B')$

$= A \cap U$

$= A$ = R.H.S

(ii) LHS $A \cup (B - A) = A \cup (B \cap A')$

$= (A \cup B) \cap (A \cup A')$ [distributive Property]

$= (A \cup B) \cap (U)$

$= A \cup B$ = RHS

Hence Proved

8) For any sets A, B and C, Prove that

i) $A-(B\cup C) = (A-B)\cap(A-C)$

ii) $A-(B\cap C) = (A-B)\cup(A-C)$

iii) $(A\cup B)-C = (A-C)\cup(B-C)$

iv) $(A\cap B)-C = (A-C)\cap(B-C)$

(ii) Practice Yourself

(iii) Practice yourself (Hint. $(A\cup B)-C$
$= (A\cup B)\cap C'$
$= (A\cap C')\cup(B\cap C')$
$= (A-C)\cup(B-C)$

✓(iv) LHS $(A\cap B)-C$

$= (A\cap B)\cap C'$

$= (A\cap C')\cap(B\cap C')$

$= (A-C)\cap(B-C) =$ RHS.

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9) For any sets A and B, Prove that
$P(A\cap B) = P(A)\cap P(B)$ where $P \to$ Powerset

Proof Let $X \in P(A\cap B)$

case-1

$\Rightarrow X \subseteq A\cap B$

$\Rightarrow X \subseteq A$ and $X \subseteq B$

$\Rightarrow X \in P(A)$ and $X \in P(B)$

$\Rightarrow X \in P(A)\cap P(B)$

$\Rightarrow P(A\cap B) \subseteq P(A)\cap P(B)$ (1)

Case-2 Let $Y \in P(A)\cap P(B)$

$\Rightarrow Y \in P(A)$ and $Y \in P(B)$

$\Rightarrow Y \subseteq A$ and $Y \subseteq B$

$\Rightarrow Y \subseteq A\cap B$

$\Rightarrow Y \in P(A\cap B)$

$\Rightarrow P(A)\cap P(B) \subseteq P(A\cap B)$ (2)

from (1) & (2)

$P(A\cap B) = P(A)\cap P(B)$

(10) If $P(A) = P(B)$ then $A = B$ (7)

Proof Case-1 To Prove $A \subseteq B$

Proof We know $A \subseteq A$ (Every set is subset of itself)

$\Rightarrow A \in P(A)$ (Subset is an element of Powerset)

$\Rightarrow A \in P(B)$ (given)

$\Rightarrow A \subseteq B$ —— (1)

Case-2 To Prove $B \subseteq A$

Proof Also $B \subseteq B$

$\Rightarrow B \in P(B)$

$\Rightarrow B \in P(A)$

$\Rightarrow B \subseteq A$ —— (2)

from (1) & (2)

$A = B$

Ques 11 If $A \cap B' = \phi$, then Prove that $A = A \cap B$, Hence show that $A \subseteq B$

Sol We can write A as

$A = A \cap U$ $U \rightarrow$ universal set

$= A \cap (B \cup B')$ (Introduced B)

$= (A \cap B) \cup (A \cap B')$ (distributive prop.)

$= (A \cap B) \cup \phi$ (given)

$= A \cap B$

$\Rightarrow A = A \cap B$ (*), Hence proved

~~Here~~ Now To Prove $A \subseteq B$

Proof Let $x \in A$

$\Rightarrow x \in A \cap B$ (from above)

$\rightarrow x \in A$ and $x \in B$

$\Rightarrow x \in B$ (Surely)

$\Rightarrow A \subseteq B$ Hence proved

Prove That $A-(B\cup C) = (A-B)\cap(A-C)$

$A-(B\cup C) = A\cap(B\cup C)'$

$= A\cap(B'\cap C')$ [de Morgan's law]

$= (A\cap A)\cap(B'\cap C')$ [$\because A = A\cap A$]

$= A\cap A\cap B'\cap C'$ [Remove all brackets

$= A\cap(A\cap B')\cap C'$ [group A & B']

$= A\cap(B'\cap A)\cap C'$ [commutative]

$= A\cap B'\cap A\cap C'$ [Remove all brackets

$= (A\cap B')\cap(A\cap C')$ [group again]

$= (A-B)\cap(A-C)$

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ASSIGNMENT NO: 1

SETS (SUBJECTIVE)

1. **Describe the following sets in roster form :**

   a) $A = \{x : x \in Z \text{ and } x^2 \leq 25\}$

   b) $B = \left\{x : x \in Z \text{ and} -\frac{11}{2} \leq x \leq \frac{11}{2}\right\}$

   c) $C = \left\{x : x = \frac{n}{n^2+1}, 1 \leq n \leq 5, n \in N\right\}$

   d) $D = \left\{x : x \textit{ positive } \text{integer } \textit{less than } 10 \textit{ and } \left(2^x - 1\right) \textit{is an odd number}\right\}$

   e) $E = \{x : x^2 + 7x - 8 = 0, x \in R\}$

2. **Describe the following sets in set builder form :**

   a) $A = \left\{0, \frac{1}{6}, \frac{2}{7}, \frac{3}{8}, \frac{4}{9}\right\}$

   b) $B = \left\{\frac{2}{3}, \frac{3}{8}, \frac{4}{15}, \frac{5}{24}, \frac{6}{35}, \frac{7}{48}, \frac{8}{63}\right\}$

   c) $C = \left\{1, \frac{1}{8}, \frac{1}{27}, \frac{1}{64}, \frac{1}{125}\right\}$

d) $D = \{2, 5, 10, 17, 26, 37, 50\}$

e) $E = \{15, 20, 25, 30, 35, 40, 45, 50\}$

3. Let $A = \left\{ x : \dfrac{x+5}{x-7} - 5 = \dfrac{4x-40}{13-x} \right\}$ **Is** A **an empty set? Justify your answer.**

4. **If** $A=\{1,2,5,8,11,12,19\}$ **,** $B=\{3,5,8,30,45\}$ **and** $C=\{1,5,8\}$ **,then verify the following results.**

i) $A-(B\cup C)=(A-B)\cap(A-C)$

ii) $A\cap(B-C)=(A\cap B)-(A\cap C)$

5. $If\ U = set\ of\ all\ digits\ in\ our\ decimal\ system, A = \{x: x\ is\ prime\}$**and** $B = \{x: x\ is\ a\ perfect\ square\}$**,then verify that**

i) $(A\cup B)' = A'\cap B'$

ii) $(A\cap B)' = A'\cup B'$

6. If $n(A \cup B) = 15, n(A) = 8 \text{ and } n(B) = 12$ .Use Venn diagram to find the following

   i) $n(A')$

   ii) $n(B')$

   iii) $n(A \cap B')$

   iv) $n(A' \cap B)$

7. A survey was conducted on 100 customers and it was found that 65 liked product X and 38 customers liked product. Find the least number of customers that must have liked both products.

8. In a survey it was found that 21 people liked product A , 26 liked B and 29 liked product C .If 14 people liked products A and B , 12 people liked products C and A , 14 people liked products B and C and 8 liked all the three products , find how many liked product C only.

9. Out of 100 students of class XI ,40 participated in a rally , 15 participated in street play,20 participated in group song , 5 participated in both rally and street play , 3 participated in both street play and group song , 1 participated in rally and group song , none of them participated in all three activities . Find the number of students who did not participate in any of these activities.

(DAV BOARD 2017)

10. For any two sets A and B , Prove that P(A) = P(B)$\Rightarrow$A=B where P(A) and P(B) denote power sets of A and B respectively. How many elements are there in P(P($\emptyset$))?

(DAV BOARD 2017)

11. For any three sets A,B and C , show that If $A \subset B$ then $C-B \subset C-A$

(DAV BOARD 2018)

12. In a group of students, half the number of students know Hindi, $\frac{2}{3}$ of them know English, 10 know both the languages and 6 students do not know either Hindi or English. Find how many students are there in the group. (DAV BOARD 2018)

13. Given that $A=\{2x : x \in N, 1 \le x < 4\}$

$B=\{x+2 : x \in N, 2 \le x < 6\}$.

Find $B-A$ **(DAV BOARD 2019)**

14. Using properties of sets, prove that for all sets A and B

$(A \cap B) \cup (A-B) = A$ **(DAV BOARD 2019)**

15. Show that $A \cup B = A \cap B$ **implies** $A = B$ **(DAV BOARD 2019)**

16. In a group of 50 students, 14 drinks orange juice but not eat apple, 30 drink orange juice and each student like at least one of them. Find

- **i. How many drinks orange juice as well as eat apple?**
- **ii. How many eat apple but not drink orange juice? (DAV BOARD 2019)**

17. From 50 students taking examinations in Mathematics, Physics and Chemistry, each of the student has passed in atleast one of the subject, 37 passed Mathematics, 24 Physics and 43 Chemistry. Atmost 19 passed Mathematics and Physics, at most 29 Mathematics and Chemistry and atmost 20 Physics and Chemistry, What is the largest possible number that could have passed all the three examination? (DAV BOARD 2020)

ASSIGNMENT NO: 1

SETS (OBJECTIVE)

Q1. The set $(A \cup B \cup C) \cap (A \cap B' \cap C') \cap C'$ is equal to

a) $B \cap C'$ b) $A \cap$ c) $B \cup C'$ d) $A \cap C'$

Q2. Let $A_1, A_2, \ldots \ldots \ldots . A_{30}$ are thirty sets each having 5 elements and $B_1, B_2 \ldots . . B_n$ are n sets each with 3 elements. Let $\bigcup_{i=1}^{30} A_i = \bigcup_{j=1}^{n} B_j = S$ and each element of S belongs to exactly 10 of the $A_i's$ and exactly 9 of the $B_j's$, then value of n is

a) 15 b) 3 c) 45 d) 35

Q3. Two finite sets have m and n elements.The number of subsets of first set is 112 more than that of the second set.The values of m and n are respectively :

a) 4 ,7 b) 7, 4 c) 4,4 d) 7,7

Q4. The set $(A \cap B')' \cup (B \cap C)$ is equal to

a) $A' \cup B \cup$ b) $A' \cup B$ c) $A' \cup C'$ d) $A' \cap B$

Q5. In a class of 60 students, 25 students play cricket and 20 students play tennis and 10 students play both the games. Then, the number of students who play neither is

a) 0 b) 25 c) 35 d) 45

Q6. A survey shows that 63% of the people watch a news channel whereas 76% watch another channel. If x % of the people watch both channel, then

a) x = 35 b) x = 63 c) 39$\leq x \leq 63$ d) x = 39

Q7. In a town of 840 persons,450 person read Hindi, 300 read English and 200 read both. Then the number of persons who read neither is

a) 210 b) 290 c) 180 d) 260

Q8. Let S=$\{x: x \text{ is a positive multiple of } 3 \text{ less than } 100\}$ and P=$\{x: x \text{ is a prime number less than } 20\}$.Then n(S) + n(P) is

a) 34 b) 31 c) 33 d) 30

Q9. If A and B are finite sets such that A ⊂ B, then n (A U B) = __________

Q10. If A and B are any two sets, then A - B = ______________

Q11. If X = { 8^n-7n -1, n∈ N} , Y = {49n - 49, n∈ N } , then

a) X ⊂ Y b) X ⊆ Y c) X ∩ Y= ϕ d) X = Y

Q12. If A and B are two set, then A∩ (A∪B) equals

a) A b) B c) ϕ d) A ∩ B

Q13. If n(A) = 115, n (B) = 326 and n(A – B) = 47, then n(A∪B) =

a)135 b)142 c) 373 d) 345

(DAV BOARD 2020)

Q14. If A = { x: $x \in R$, $x < 5$ } and B = { x: $x \in R$, $x > 4$ }, then $A \cap B$

a) (3,5) **b) (4,2)** **c) (4,5)** **d) (5,4)**

Q15. Given two finite sets, A and B such that n(A) = 3 and n(B) = 6. The minimum number of elements in A U B is

a) 3 **b) 6** **c) 9** **d) 18**

(DAV BOARD 2020)