DAV INSTITUTIONS, ODISHA ZONE- I
D.A.V. PUBLIC SCHOOL, BERHMAPUR, ODISHA

SUBJECT : MATHEMATICS
CLASS : XII
TOPIC : DEFINITE INTEGRAL

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WORKSHEET – 3 (ADVANCE)

SECTION – A (Each question carry 1 mark)

1. $\int_{a+c}^{b+c} f(x)dx$ is equal to

   (A) $\int_a^b f(x-c)dx$ (B) $\int_a^b f(x+c)dx$ (C) $\int_a^b f(x)dx$ (D) $\int_{a-c}^{b-c} f(x)dx$.

2. If $\int_0^a \frac{1}{1+4x^2}dx = \frac{\pi}{8}$ $, then\ a =$ -------------------.

SECTION – B (Each question carry 2 mark)

3. Evaluate : $\int_{-1}^{1} e^{|x|}dx$.

4. Prove that : $\int_a^b \frac{f(x)}{f(x)+f(a+b-x)}dx = \frac{b-a}{2}$.

SECTION – C (Each question carry 4 mark)

5. Find the value of the integral $\int_0^1 \frac{\sqrt{1-x}}{1+x}dx$.

6. Find $\int_0^{1000} e^{x-[x]}dx$.

7. Evaluate : $\int_0^{\frac{\pi}{2}} log\left(\frac{4+3sinx}{4+3cosx}\right)dx$.

8. Evaluate : $\int_0^{\pi} \frac{x}{1+cos\alpha\ .sinx}dx$.

SECTION – D (Each question carry 6 mark)

9. Evaluate : $\int_0^{1.5}[x^2]dx$.

10. Evaluate : $\int_0^1 \cot^{-1}(1-x+x^2)dx$.

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MARKING SCHEME (SAMPLE)

Qsn No.	VALUE POINTS	MARK FOR EACH VALUE POINT	MAX. MARK
1	**Evaluate :** $\int_2^3 3^x dx$. **Solution :** Now $\int_2^3 3^x dx = \left[\frac{3^x}{\log_e 3}\right]_2^3$ $= \frac{3^3}{\log_e 3} - \frac{3^2}{\log_e 3} = \log_3 e^{18} = 18 \log_3 e$.	½ ½	1
2	**Evaluate :** $\int_{-1}^{2} \frac{\lvert x\rvert}{x} dx$. **Solution :** We have $\lvert x\rvert = \begin{cases} x, & for\ x \geq 0 \\ -x, & for\ x < 0 \end{cases}$ Now $\int_{-1}^{2} \frac{\lvert x\rvert}{x} dx = \int_{-1}^{0} \frac{-x}{x} dx + \int_0^2 \frac{x}{x} dx$ $= -\int_{-1}^{0} 1dx + \int_0^2 1dx$ $= [-x]_{-1}^{0} + [x]_0^2$ $= -1 + 2 = 1$	½ ½ ½ ½	2
3	**Evaluate :** $\int_0^\pi \frac{xsinx}{1+\cos^2 x} dx$ **Solution :** Let $I = \int_0^\pi \frac{xsinx}{1+\cos^2 x} dx$ $\Rightarrow I = \int_0^\pi \frac{(\pi - x)sinx}{1+\cos^2 x} dx$ [using prop. $\int_0^a f(x) = \int_0^a f(a-x)dx$] $\Rightarrow 2I = \pi \int_0^\pi \frac{sinx}{1+\cos^2 x} dx$ $Let\ cosx = t \Rightarrow -sinxdx = dt$ $when\ x = 0, t = 1$ $when\ x = \pi, t = -1$	½ ½ ½	4

Qsn No.	VALUE POINTS	MARK FOR EACH VALUE POINT	MAX. MARK
	$\Rightarrow 2I = -\pi \int_1^{-1} \frac{1}{1+t^2} dt.$ $= \pi[\tan^{-1} t]_1^{-1} = -\pi[\tan^{-1}(-1) - \tan^{-1}(1)]$ $\Rightarrow 2I = -\pi\left[-\frac{\pi}{4} - \frac{\pi}{4}\right] = \frac{\pi^2}{2}$ $\Rightarrow I = \frac{\pi^2}{4}$	½ 1 1	
4	**Evaluate the following definite integral as limit of sums :** $\int_0^4 (3x^2 + 2x + 1)dx$ **.** **Solution :** Here $f(x) = 3x^2 + 2x + 1, a = 0, b = 4, h = \frac{4-0}{n}$ $\Rightarrow nh = 4$ $\therefore \int_0^4 (3x^2 + 2x + 1)dx$ $= \lim_{h\to 0} h[f(h) + f(2h) + \; \ldots . + f(nh)]$ $= \lim_{h\to 0} h[(3h^2 + 2h + 1) + (12h^2 + 4h + 1) +$ *....+3n2h2+2nh+1]* $= \lim_{h\to 0} h[(3h^2 + 12h^2 + \; \ldots . + 3n^2h^2) + (2h + 4h$ $+ \; \ldots\ldots + 2nh) + n]$ $= \lim_{h\to 0} h[3h^2(1^2 + 2^2 + \; \ldots\ldots + n^2) + 2h(1 + 2 +$ *...+n+n]* $= \lim_{h\to 0}[3h^3 \frac{n(n+1)(2n+1)}{6} + 2h^2 \frac{n(n+1)}{2} \quad + nh]$ $= \lim_{h\to 0}[\frac{nh(nh + h)(2nh + h)}{2} + nh(nh + h) + nh]$ $= \frac{4(4)(8)}{2} + 4(4) + 4 = 64 + 16 + 4 = 84$	1 1 1 1 1 1	6

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