13.6 Volume of a Cuboid

You have already learnt about volumes of certain figures (objects) in earlier classes. Recall that solid objects occupy space. The measure of this occupied space is called the **Volume** of the object.

Note : If an object is solid, then the space occupied by such an object is measured, and is termed the **Volume** of the object. On the other hand, if the object is hollow, then interior is empty, and can be filled with air, or some liquid that will take the shape of its container. In this case, the volume of the substance that can fill the interior is called the **capacity of the container**. In short, the volume of an object is the measure of the space it occupies, and the capacity of an object is the volume of substance its interior can accommodate. Hence, the unit of measurement of either of the two is cubic unit.

So, if we were to talk of the volume of a cuboid, we would be considering the measure of the space occupied by the cuboid.

Further, the area or the volume is measured as the magnitude of a region. So, correctly speaking, we should be finding the area of a circular region, or volume of a cuboidal region, or volume of a spherical region, etc. But for the sake of simplicity, we say, find the area of a circle, volume of a cuboid or a sphere even though these mean only their boundaries.

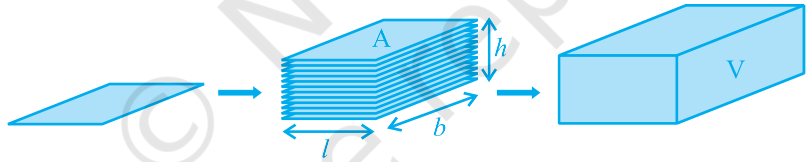

Fig. 13.23

Observe Fig. 13.23. Suppose we say that the area of each rectangle is A, the height up to which the rectangles are stacked is h and the volume of the cuboid is V. Can you tell what would be the relationship between V, A and h?

The area of the plane region occupied by each rectangle × height

= Measure of the space occupied by the cuboid

So, we get $A \times h = V$

That is, $\boxed{\textbf{Volume of a Cuboid = base area × height = length × breadth × height}}$

or $l \times b \times h$, where l, b and h are respectively the length, breadth and height of the cuboid.

Note : When we measure the magnitude of the region of a space, that is, the space occupied by a solid, we do so by counting the number of cubes of edge of unit length that can fit into it exactly. Therefore, the unit of measurement of volume is cubic unit.

Again $\boxed{\textbf{Volume of a Cube} = \textbf{edge} \times \textbf{edge} \times \textbf{edge} = a^3}$

where a is the edge of the cube (see Fig. 13.24).

So, if a cube has edge of 12 cm,

then volume of the cube = $12 \times 12 \times 12$ cm^3

= 1728 cm^3.

Recall that you have learnt these formulae in earlier classes. Now let us take some examples to illustrate the use of these formulae:

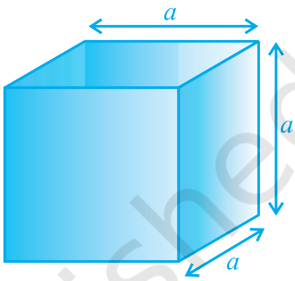

Fig. 13.24

Example11 : A wall of length 10 m was to be built across an open ground. The height of the wall is 4 m and thickness of the wall is 24 cm. If this wall is to be built up with bricks whose dimensions are 24 cm × 12 cm × 8 cm, how many bricks would be required?

Solution : Since the wall with all its bricks makes up the space occupied by it, we need to find the volume of the wall, which is nothing but a cuboid.

Here, Length = 10 m = 1000 cm

Thickness = 24 cm

Height = 4 m = 400 cm

Therefore, Volume of the wall = length × thickness × height

= $1000 \times 24 \times 400$ cm^3

Now, each brick is a cuboid with length = 24 cm, breadth = 12 cm and height = 8 cm

So, volume of each brick = length × breadth × height

= $24 \times 12 \times 8$ cm^3

$$\text{So, number of bricks required} = \frac{\text{volume of the wall}}{\text{volume of each brick}}$$

$$= \frac{1000 \times 24 \times 400}{24 \times 12 \times 8}$$

$$= 4166.6$$

So, the wall requires 4167 bricks.

Example 12 : A child playing with building blocks, which are of the shape of cubes, has built a structure as shown in Fig. 13.25. If the edge of each cube is 3 cm, find the volume of the structure built by the child.

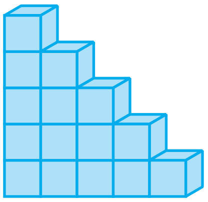

Fig. 13.25

Solution : Volume of each cube = edge × edge × edge

$= 3 \times 3 \times 3 \text{ cm}^3 = 27 \text{ cm}^3$

Number of cubes in the structure = 15

Therefore, volume of the structure = $27 \times 15 \text{ cm}^3$

$= 405 \text{ cm}^3$

EXERCISE 13.5

1. A matchbox measures 4 cm × 2.5 cm × 1.5 cm. What will be the volume of a packet containing 12 such boxes?
2. A cuboidal water tank is 6 m long, 5 m wide and 4.5 m deep. How many litres of water can it hold? ($1 \text{ m}^3 = 1000\ l$)
3. A cuboidal vessel is 10 m long and 8 m wide. How high must it be made to hold 380 cubic metres of a liquid?
4. Find the cost of digging a cuboidal pit 8 m long, 6 m broad and 3 m deep at the rate of ₹ 30 per m^3.
5. The capacity of a cuboidal tank is 50000 litres of water. Find the breadth of the tank, if its length and depth are respectively 2.5 m and 10 m.
6. A village, having a population of 4000, requires 150 litres of water per head per day. It has a tank measuring 20 m × 15 m × 6 m. For how many days will the water of this tank last?
7. A godown measures 40 m × 25 m × 15 m. Find the maximum number of wooden crates each measuring 1.5 m × 1.25 m × 0.5 m that can be stored in the godown.
8. A solid cube of side 12 cm is cut into eight cubes of equal volume. What will be the side of the new cube? Also, find the ratio between their surface areas.
9. A river 3 m deep and 40 m wide is flowing at the rate of 2 km per hour. How much water will fall into the sea in a minute?

13.7 Volume of a Cylinder

Just as a cuboid is built up with rectangles of the same size, we have seen that a right circular cylinder can be built up using circles of the same size. So, using the same argument as for a cuboid, we can see that the volume of a cylinder can be obtained as : base area × height

= area of circular base × height = $\pi r^2 h$

So, $\boxed{\textbf{Volume of a Cylinder} = \pi r^2 h}$

where r is the base radius and h is the height of the cylinder.

Example 13 : The pillars of a temple are cylindrically shaped (see Fig. 13.26). If each pillar has a circular base of radius 20 cm and height 10 m, how much concrete mixture would be required to build 14 such pillars?

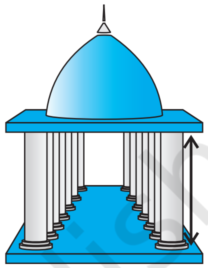

Fig. 13.26

Solution : Since the concrete mixture that is to be used to build up the pillars is going to occupy the entire space of the pillar, what we need to find here is the volume of the cylinders.

Radius of base of a cylinder = 20 cm

Height of the cylindrical pillar = 10 m = 1000 cm

So, volume of each cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times 20 \times 20 \times 1000 \text{ cm}^3$$

$$= \frac{8800000}{7} \text{ cm}^3$$

$$= \frac{8.8}{7} \text{ m}^3 \text{ (Since } 1000000 \text{ cm}^3 = 1 \text{m}^3\text{)}$$

Therefore, volume of 14 pillars = volume of each cylinder × 14

$$= \frac{8.8}{7} \times 14 \text{ m}^3$$

$$= 17.6 \text{ m}^3$$

So, 14 pillars would need 17.6 m^3 of concrete mixture.

Example 14 : At a Ramzan Mela, a stall keeper in one of the food stalls has a large cylindrical vessel of base radius 15 cm filled up to a height of 32 cm with orange juice. The juice is filled in small cylindrical glasses (see Fig. 13.27) of radius 3 cm up to a height of 8 cm, and sold for ₹ 15 each. How much money does the stall keeper receive by selling the juice completely?

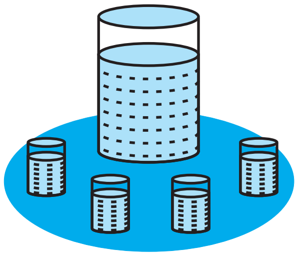

Fig. 13.27

Solution : The volume of juice in the vessel

$$= \text{volume of the cylindrical vessel}$$

$$= \pi R^2 H$$

(where R and H are taken as the radius and height respectively of the vessel)

$$= \pi \times 15 \times 15 \times 32 \text{ cm}^3$$

Similarly, the volume of juice each glass can hold = $\pi r^2 h$

(where r and h are taken as the radius and height respectively of each glass)

$$= \pi \times 3 \times 3 \times 8 \text{ cm}^3$$

So, number of glasses of juice that are sold

$$= \frac{\text{volume of the vessel}}{\text{volume of each glass}}$$

$$= \frac{\pi \times 15 \times 15 \times 32}{\pi \times 3 \times 3 \times 8}$$

$$= 100$$

Therefore, amount received by the stall keeper = ₹ 15 × 100

= ₹ 1500

EXERCISE 13.6

Assume $\pi = \frac{22}{7}$, unless stated otherwise.

1. The circumference of the base of a cylindrical vessel is 132 cm and its height is 25 cm. How many litres of water can it hold? ($1000 \text{ cm}^3 = 1l$)
2. The inner diameter of a cylindrical wooden pipe is 24 cm and its outer diameter is 28 cm. The length of the pipe is 35 cm. Find the mass of the pipe, if 1 cm^3 of wood has a mass of 0.6 g.
3. A soft drink is available in two packs – (i) a tin can with a rectangular base of length 5 cm and width 4 cm, having a height of 15 cm and (ii) a plastic cylinder with circular base of diameter 7 cm and height 10 cm. Which container has greater capacity and by how much?
4. If the lateral surface of a cylinder is 94.2 cm^2 and its height is 5 cm, then find (i) radius of its base (ii) its volume. (Use $\pi = 3.14$)

5. It costs ₹ 2200 to paint the inner curved surface of a cylindrical vessel 10 m deep. If the cost of painting is at the rate of ₹ 20 per m^2, find

(i) inner curved surface area of the vessel,

(ii) radius of the base,

(iii) capacity of the vessel.

6. The capacity of a closed cylindrical vessel of height 1 m is 15.4 litres. How many square metres of metal sheet would be needed to make it?

7. A lead pencil consists of a cylinder of wood with a solid cylinder of graphite filled in the interior. The diameter of the pencil is 7 mm and the diameter of the graphite is 1 mm. If the length of the pencil is 14 cm, find the volume of the wood and that of the graphite.

8. A patient in a hospital is given soup daily in a cylindrical bowl of diameter 7 cm. If the bowl is filled with soup to a height of 4 cm, how much soup the hospital has to prepare daily to serve 250 patients?

13.8 Volume of a Right Circular Cone

In Fig 13.28, can you see that there is a right circular cylinder and a right circular cone of the same base radius and the same height?

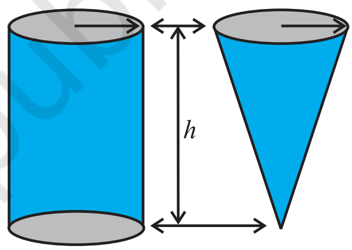

Fig. 13.28

Activity : Try to make a hollow cylinder and a hollow cone like this with the same base radius and the same height (see Fig. 13.28). Then, we can try out an experiment that will help us, to see practically what the volume of a right circular cone would be!

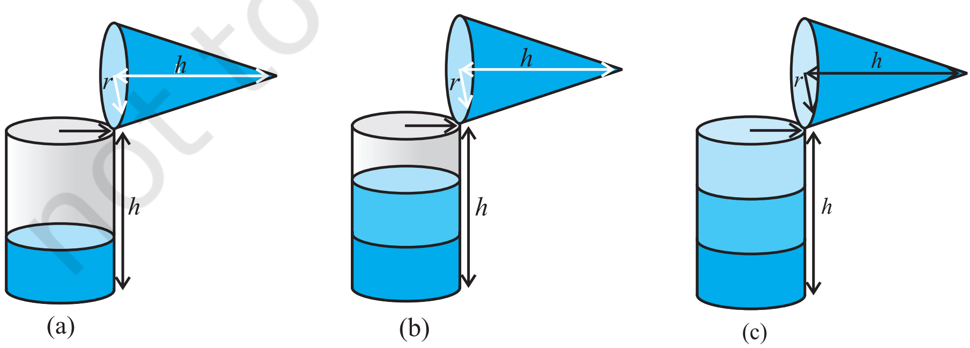

Fig. 13.29

So, let us start like this.

Fill the cone up to the brim with sand once, and empty it into the cylinder. We find that it fills up only a part of the cylinder [see Fig. 13.29(a)].

When we fill up the cone again to the brim, and empty it into the cylinder, we see that the cylinder is still not full [see Fig. 13.29(b)].

When the cone is filled up for the third time, and emptied into the cylinder, it can be seen that the cylinder is also full to the brim [see Fig. 13.29(c)].

With this, we can safely come to the conclusion that three times the volume of a cone, makes up the volume of a cylinder, which has the same base radius and the same height as the cone, which means that the volume of the cone is one-third the volume of the cylinder.

So, $$\boxed{\textbf{Volume of a Cone} = \frac{1}{3}\pi r^2 h}$$

where r is the base radius and h is the height of the cone.

Example 15 : The height and the slant height of a cone are 21 cm and 28 cm respectively. Find the volume of the cone.

Solution : From $l^2 = r^2 + h^2$, we have

$$r = \sqrt{l^2 - h^2} = \sqrt{28^2 - 21^2} \text{ cm} = 7\sqrt{7} \text{ cm}$$

So, volume of the cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 7\sqrt{7} \times 7\sqrt{7} \times 21 \text{ cm}^3$

$= 7546 \text{ cm}^3$

Example 16 : Monica has a piece of canvas whose area is 551 m^2. She uses it to have a conical tent made, with a base radius of 7 m. Assuming that all the stitching margins and the wastage incurred while cutting, amounts to approximately 1 m^2, find the volume of the tent that can be made with it.

Solution : Since the area of the canvas = 551 m^2 and area of the canvas lost in wastage is 1 m^2, therefore the area of canvas available for making the tent is (551 – 1) m^2 = 550 m^2.

Now, the surface area of the tent = 550 m^2 and the required base radius of the conical tent = 7 m

Note that a tent has only a curved surface (the floor of a tent is not covered by canvas!!).

Therefore, curved surface area of tent = 550 m^2.

That is, $\pi r l = 550$

or, $\frac{22}{7} \times 7 \times l = 550$

or, $l = 3\frac{550}{22} \text{ m} = 25 \text{ m}$

Now, $l^2 = r^2 + h^2$

Therefore, $h = \sqrt{l^2 - r^2} = \sqrt{25^2 - 7^2} \text{ m} = \sqrt{625 - 49} \text{ m} = \sqrt{576} \text{ m}$

$= 24 \text{ m}$

So, the volume of the conical tent $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 24 \text{ m}^3 = 1232 \text{ m}^3$.

EXERCISE 13.7

Assume $\pi = \frac{22}{7}$, unless stated otherwise.

1. Find the volume of the right circular cone with
(i) radius 6 cm, height 7 cm (ii) radius 3.5 cm, height 12 cm

2. Find the capacity in litres of a conical vessel with
(i) radius 7 cm, slant height 25 cm (ii) height 12 cm, slant height 13 cm

3. The height of a cone is 15 cm. If its volume is 1570 cm^3, find the radius of the base. (Use $\pi = 3.14$)

4. If the volume of a right circular cone of height 9 cm is 48 π cm^3, find the diameter of its base.

5. A conical pit of top diameter 3.5 m is 12 m deep. What is its capacity in kilolitres?

6. The volume of a right circular cone is 9856 cm^3. If the diameter of the base is 28 cm, find
(i) height of the cone (ii) slant height of the cone
(iii) curved surface area of the cone

7. A right triangle ABC with sides 5 cm, 12 cm and 13 cm is revolved about the side 12 cm. Find the volume of the solid so obtained.

8. If the triangle ABC in the Question 7 above is revolved about the side 5 cm, then find the volume of the solid so obtained. Find also the ratio of the volumes of the two solids obtained in Questions 7 and 8.

9. A heap of wheat is in the form of a cone whose diameter is 10.5 m and height is 3 m. Find its volume. The heap is to be covered by canvas to protect it from rain. Find the area of the canvas required.

13.9 Volume of a Sphere

Now, let us see how to go about measuring the volume of a sphere. First, take two or three spheres of different radii, and a container big enough to be able to put each of the spheres into it, one at a time. Also, take a large trough in which you can place the container. Then, fill the container up to the brim with water [see Fig. 13.30(a)].

Now, carefully place one of the spheres in the container. Some of the water from the container will over flow into the trough in which it is kept [see Fig. 13.30(b)]. Carefully pour out the water from the trough into a measuring cylinder (i.e., a graduated cylindrical jar) and measure the water over flowed [see Fig. 13.30(c)]. Suppose the radius of the immersed sphere is r (you can find the radius by measuring the diameter of the sphere). Then evaluate $\frac{4}{3}\pi r^3$. Do you find this value almost equal to the measure of the volume over flowed?

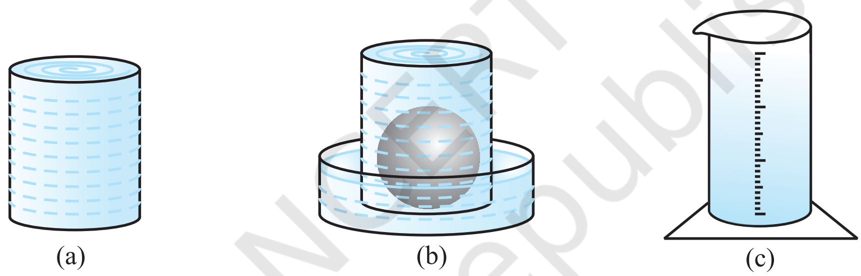

Fig. 13.30

Once again repeat the procedure done just now, with a different size of sphere. Find the radius R of this sphere and then calculate the value of $\frac{4}{3}\pi R^3$. Once again this value is nearly equal to the measure of the volume of the water displaced (over flowed) by the sphere. What does this tell us? We know that the volume of the sphere is the same as the measure of the volume of the water displaced by it. By doing this experiment repeatedly with spheres of varying radii, we are getting the same result, namely, the volume of a sphere is equal to $\frac{4}{3}\pi$ times the cube of its radius. This gives us the idea that

$$\boxed{\textbf{Volume of a Sphere} = \frac{4}{3}\pi r^3}$$

where r is the radius of the sphere.

Later, in higher classes it can be proved also. But at this stage, we will just take it as true.

Since a hemisphere is half of a sphere, can you guess what the volume of a hemisphere will be? Yes, it is $\frac{1}{2}$ of $\frac{4}{3}\pi r^3 = \frac{2}{3}\pi r^3$.

So, $\boxed{\textbf{Volume of a Hemisphere} = \frac{2}{3}\pi r^3}$

where r is the radius of the hemisphere.

Let us take some examples to illustrate the use of these formulae.

Example 17 : Find the volume of a sphere of radius 11.2 cm.

Solution : Required volume $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times 11.2 \times 11.2 \times 11.2 \text{ cm}^3 = 5887.32 \text{ cm}^3$$

Example 18 : A shot-putt is a metallic sphere of radius 4.9 cm. If the density of the metal is 7.8 g per cm^3, find the mass of the shot-putt.

Solution : Since the shot-putt is a solid sphere made of metal and its mass is equal to the product of its volume and density, we need to find the volume of the sphere.

Now, volume of the sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times 4.9 \times 4.9 \times 4.9 \text{ cm}^3$$

$$= 493 \text{ cm}^3 \text{ (nearly)}$$

Further, mass of 1 cm^3 of metal is 7.8 g.

Therefore, mass of the shot-putt = 7.8 × 493 g

$$= 3845.44 \text{ g} = 3.85 \text{ kg (nearly)}$$

Example 19 : A hemispherical bowl has a radius of 3.5 cm. What would be the volume of water it would contain?

Solution : The volume of water the bowl can contain

$$= \frac{2}{3}\pi r^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 3.5 \text{ cm}^3 = 89.8 \text{ cm}^3$$

EXERCISE 13.8

Assume $\pi = \frac{22}{7}$, unless stated otherwise.

1. Find the volume of a sphere whose radius is
(i) 7 cm (ii) 0.63 m

2. Find the amount of water displaced by a solid spherical ball of diameter
(i) 28 cm (ii) 0.21 m

3. The diameter of a metallic ball is 4.2 cm. What is the mass of the ball, if the density of the metal is 8.9 g per cm^3?

4. The diameter of the moon is approximately one-fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?

5. How many litres of milk can a hemispherical bowl of diameter 10.5 cm hold?

6. A hemispherical tank is made up of an iron sheet 1 cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank.

7. Find the volume of a sphere whose surface area is 154 cm^2.

8. A dome of a building is in the form of a hemisphere. From inside, it was white-washed at the cost of ₹ 4989.60. If the cost of white-washing is ₹ 20 per square metre, find the
(i) inside surface area of the dome, (ii) volume of the air inside the dome.

9. Twenty seven solid iron spheres, each of radius r and surface area S are melted to form a sphere with surface area S′. Find the
(i) radius r' of the new sphere, (ii) ratio of S and S′.

10. A capsule of medicine is in the shape of a sphere of diameter 3.5 mm. How much medicine (in mm^3) is needed to fill this capsule?

EXERCISE 13.9 (Optional)*

1. A wooden bookshelf has external dimensions as follows: Height = 110 cm, Depth = 25 cm, Breadth = 85 cm (see Fig. 13.31). The thickness of the plank is 5 cm everywhere. The external faces are to be polished and the inner faces are to be painted. If the rate of polishing is 20 paise per cm^2 and the rate of painting is 10 paise per cm^2, find the total expenses required for polishing and painting the surface of the bookshelf.

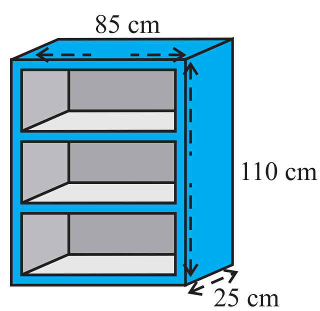

Fig. 13.31

*These exercises are not from examination point of view.

2. The front compound wall of a house is decorated by wooden spheres of diameter 21 cm, placed on small supports as shown in Fig 13.32. Eight such spheres are used for this purpose, and are to be painted silver. Each support is a cylinder of radius 1.5 cm and height 7 cm and is to be painted black. Find the cost of paint required if silver paint costs 25 paise per cm^2 and black paint costs 5 paise per cm^2.

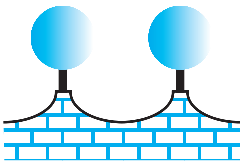

Fig. 13.32

3. The diameter of a sphere is decreased by 25%. By what per cent does its curved surface area decrease?

13.10 Summary

In this chapter, you have studied the following points:

1. **Surface area of a cuboid = $2(lb + bh + hl)$**
2. **Surface area of a cube = $6a^2$**
3. **Curved surface area of a cylinder = $2\pi rh$**
4. **Total surface area of a cylinder = $2\pi r(r + h)$**
5. **Curved surface area of a cone = πrl**
6. **Total surface area of a right circular cone = $\pi rl + \pi r^2$, i.e., $\pi r(l + r)$**
7. **Surface area of a sphere of radius $r = 4\pi r^2$**
8. **Curved surface area of a hemisphere = $2\pi r^2$**
9. **Total surface area of a hemisphere = $3\pi r^2$**
10. **Volume of a cuboid = $l \times b \times h$**
11. **Volume of a cube = a^3**
12. **Volume of a cylinder = $\pi r^2 h$**
13. **Volume of a cone = $\frac{1}{3}\pi r^2 h$**
14. **Volume of a sphere of radius $r = \frac{4}{3}\pi r^3$**
15. **Volume of a hemisphere = $\frac{2}{3}\pi r^3$**

[Here, letters l, b, h, a, r, etc. have been used in their usual meaning, depending on the context.]