

HOTS

LA-1 type Questions

1. Let $f: R \rightarrow R$ and $g: R \rightarrow R$ be two functions defined by $f(x) = |x| + x$ and $f(x) = |x| - x$. Find fog and gof.
2. Let $f: R \rightarrow R$ be a function given by $f(x) = px + q$. Find the constants p,q such that $f \circ f = I_R$.
3. Show that $f: R \rightarrow (-1,1)$ is defined by $f(x) = \frac{10^x - 10^{-x}}{10^x + 10^{-x}}$ is invertible. Also find $f^{-1}(x)$.
4. Let $f: R \rightarrow R$ be a function given by $f(x) = \frac{x}{x^2+1}$, $x \in R$ is neither one-one nor onto.
5. If $f: R \rightarrow R$ is given by $f(x) = \sin^2 x + \sin^2 \left(x + \frac{\pi}{3}\right) + \cos x \cdot \cos \left(x + \frac{\pi}{3}\right)$ and $g: R \rightarrow R$ is such that $g\left(\frac{5}{4}\right) = 1$. Show that gof is a constant function.

LA-2 type Questions

6. If $f: [0, \infty) \rightarrow [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(y) = \frac{\sqrt{y+6}-1}{3}$. Hence find, $f^{-1}(3)$ and (ii) y, if $3f^{-1}(y) = 4$.
7. Consider $f: R^+ \rightarrow [-9, \infty)$ given by $f(x) = 5x^2 + 6x - 9$. Prove that f is invertible with $f^{-1}(y) = \left(\frac{\sqrt{54+5y}-3}{5}\right)$
8. Let $A = R - \{3\}$, $B = R - \{1\}$. Let $f: A \rightarrow B$ be defined by $f(x) = \frac{x-2}{x-3} \quad \forall x \in A$. Show that f is bijective. Find inverse of f. Also find $f^{-1}(5)$.
9. Let $f: W \rightarrow W$ be defined as $f(n) = \begin{cases} n-1 & \text{if } n \text{ is odd} \\ n+1 & \text{if } n \text{ is even} \end{cases}$. Show that f is invertible. Find the inverse of f. Here W is the set of all whole numbers.
10. If $e^{f(x)} = \frac{10+x}{10-x}$, $x \in (-10,10)$ and $f(x) = k f\left(\frac{200x}{100+x^2}\right)$, then find the value of 'k'.