

BASIC

Multiple Choice Questions

1. If A has 5 elements and B has 7 elements then the number of one-one function from A to B is
(a) $C(7, 5)$ (b) $C(7, 5) \times 5!$ (c) 5^7 (d) 7^5
2. Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{a, b\}$. Then the number of surjections from A to B is
(a) 25 (b) 30 (c) 32 (d) 23
3. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be given by $f(x) = (8 - x^3)^{1/3}$, then $f \circ f(x)$ is
(a) $x^{1/3}$ (b) x^3 (c) x (d) $3 - x^3$
4. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = \sin x$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ defined by $g(x) = x^2$, then $f \circ g$ is
(a) $x^2 \sin x$ (b) $(\sin x)^2$ (c) $\sin x^2$ (d) $\frac{\sin x}{x^2}$
5. Let $f: A \rightarrow B$ and $g: B \rightarrow C$ be the injective function, then $(g \circ f)^{-1}$ is
(a) $f^{-1} \circ g^{-1}$ (b) $f \circ g$ (c) $g^{-1} \circ f^{-1}$ (d) $g \circ f$

Fill in the blanks

6. If $f(x) = x + 7$ and $g(x) = x - 7, x \in \mathbb{R}$, then $f \circ g(6)$ is _____.
7. If f is an invertible function given by $f(x) = \frac{3x-4}{5}$, then $f^{-1}(x) = \underline{\hspace{2cm}}$.
8. If $f(x) = 8x^3$ and $g(x) = x^{1/3}$, then $f \circ g(x) = \underline{\hspace{2cm}}$.
9. If $n(A) = m = n(B)$, then number of bijections from A to B is _____.
10. If $f(x) = \frac{x}{\sqrt{1+x^2}}$, then $(f \circ f)(x)$ is _____.

Answer the followings in a word or in a sentence

11. If $f(x) = \text{sgn}(x)$ and $g(x) = [x]$ then find $(f \circ g)(x)$ for $x \in (0, 1)$.
12. If $n(A) = m, n(B) = n$ and $f: A \rightarrow B$ is a bijection then what is the relation between m and n .
13. If $f = \{(1, b), (2, a), (3, c)\}$ and $g = \{(a, x), (b, z), (c, y)\}$, then find $g \circ f$.
14. If $f: \mathbb{R} \rightarrow \mathbb{R}$, such that $f(x) = x^3 + 5$ find $f^{-1}(x)$.
15. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = \sin x$. State whether 'f' is a bijection.

SHORT ANSWER TYPE QUESTIONS

1. Check whether $f: R \rightarrow R$ defined by $f(x) = |x|$ is a bijection .
2. Let $f: R \rightarrow R$ defined by $f(x) = x^2 + 1$.
Find the pre-images of 17 and -3 .
3. If $g = \{(1,1), (2,3), (3,5), (4,7)\}$ and $g(x) = \alpha x + \beta$ then find α, β .
4. Let $f: [0,1] \rightarrow [0,1]$ defined by $f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ 1-x & \text{if } x \text{ is irrational} \end{cases}$.
Find $(f \circ f)(x)$.
5. If $f(x) = |x|$ and $g(x) = [x]$, then find $(f \circ g)\left(-\frac{2}{3}\right) - (g \circ f)\left(-\frac{2}{3}\right)$.
6. If $g(f(x)) = |\sin x|$ and $f(g(x)) = (\sin \sqrt{x})^2$, then find $f(x)$ and $g(x)$.
7. If $f: R \rightarrow R$ defined by $f(x) = 3x^2 - 5$ and $g: R \rightarrow R$ by $g(x) = \frac{x}{x^2+1}$,
then find $g \circ f$.
8. Let C be the set of complex numbers . Prove that the mapping $f: C \rightarrow R$
is not one-one .
9. Suppose $X = \{1, 2, 3\}$ and $Y = \{4, 5, 6\}$. Find whether $f = \{(1,4), (2,6)\}$ is one-one.
10. If $f: R - \left\{\frac{3}{5}\right\} \rightarrow R$ defined by $f(x) = \frac{3x+2}{5x-3}$.
Then prove that $f^{-1}(x) = f(x)$.
11. Write $f \circ g$, if $f: R \rightarrow R$ and $g: R \rightarrow R$ is given by $f(x) = |x|$ and
 $g(x) = |5x - 1|$, for $x > 0$.
12. If $f(x) = \{4 - (x - 7)^3\}$, then find $f^{-1}(x)$.
13. If $f: [1, \infty) \rightarrow [2, \infty)$ is defined by $f(x) = x + \frac{1}{x}$, then find $f^{-1}(x)$.
14. Let $f(x) = \frac{\alpha x}{x+1}$, $x \neq -1$. Then for what value of α is $f(f(x)) = x$?
15. If $f(x) = \sin^2 x$ and the composite function $g(f(x)) = |\sin x|$, then find
 $g(x)$.