DAV PUBLIC SCHOOL, UNIT-VIII
SUB-MATHEMATICS

CHAPTER:-DETERMINANTS

WORKSHEET (ADVANCE)

CLASS:-XII

1. If $m \in N$ and $m \geq 2$, prove that $\begin{vmatrix} 1 & 1 & 1 \\ {}^{m}C_1 & {}^{m+1}C_1 & {}^{m+2}C_1 \\ {}^{m}C_2 & {}^{m+1}C_2 & {}^{m+2}C_2 \end{vmatrix} = 1$.

2. If a, b, c re the roots of the equations $x^3 + px + q = 0$, then find the value of the determinant
$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$.

3. If a, b, c are real numbers, prove that $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = (a+b+c)(a+b\omega+c\omega^2)(a+b\omega^2+c\omega)$; where ω is a complex root of unity.

4. Let a, b, c be positive real numbers. Then solve the following system of equations, if solution exists
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$$
$$\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$
$$-\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

5. If $x+y+z=0$, prove that $\begin{vmatrix} xa & yb & zc \\ yc & za & xb \\ zb & xc & ya \end{vmatrix} = xyz\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$.

6. Let f(t) = $\begin{vmatrix} \cos t & t & 1 \\ 2\sin t & t & 2t \\ \sin t & t & t \end{vmatrix}$, then find $\lim_{t \to 0} \frac{f(t)}{t^2}$.

7. Show that $\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x)$