DAV PUBLIC SCHOOL CHANDRASEKHARPUR (ZONE-1)

SUBJECT- MATHEMATICS   CLASS XII

CHAPTER-INVERSE TRIGONOMETRY FUNCTION

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CONTENT

- Introduction
- Inverse function
- Define function
- Graph
- Domain ad Range
- Principal value and $f\left(f^{-1}(x)\right) and\ f^{-1}(f(x))$
- properties

BASIC CONCEPTS(XI AND XII)

- DOMAIN AND RANGE OF TRIGO FUNCTION

FUNCTION	DOMAIN	RANGE	PRESENTATION (BIJECTIVE MAPPING)
Sin x	R	[-1 , 1]	Sin x: R→[-1 , 1]
Cos x	R	[-1 , 1]	Cos x : R→[-1 , 1]
tan x	R-{(2n+1)π/2,n∊Z}	R	tan x: R-{(2n+1)π/2,n∊Z} →R
Cot x	R-{nπ ,n∊Z }	R	Cot x: R-{nπ ,n∊Z } →R
Sec x	R-{(2n+1)π/2,n∊Z}	R-(-1 , 1)	Sec x : R-{(2n+1)π/2,n∊Z} →R-(-1 , 1)
Cosec x	R-{nπ ,n∊Z }	R-(-1 , 1)	Cosec x : R-{nπ ,n∊Z } →R-(-1 , 1)

BASICS

We will discuss here about inverse trigonometric function or inverse circular function

- Inverse of the function exist if f is one-one onto (Bijective) and given by
- $$f(x) = y \leftrightarrow x = f^{-1}(y)$$

BASICS

- Consider the sine function clearly $sin\theta : R \rightarrow R$
- given by $sin\theta = x$ for all $\theta \in R$ is many one function so its inverse does not exist. If you are restrict its domain to the interval [-π/2 , π/2] then we can declare the function is one-one .
- sine of any one of these angle θ is equal to x here angle θ is represented as $sin^{-1}x$ which is read as sine inverse x or arc sinx

BASICS

- Note that difference between $sin^{-1}x$ and $sin\theta$
- $sin^{-1}x$ represents an angle while $sin\theta$ represents pure umber
- For a given value θ gives definite finite value of $sin\theta$.
- if x is a real numbers lying between [-1 ,1] then $sin^{-1}x$ is a angle between [-π/2 , π/2]

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BASICS

- Consider the sine function clearly $\sin\theta : R \to R$ given by $\sin\theta = x$ for all $\theta \in R$ is many one function so its inverse does not exist. If you are restrict its domain to the interval $[-\pi/2, \pi/2]$

then we may have infinite many values of angle θ which satisfy the equation

$\sin\theta = x$ sine of any one of this angle is equal to x

REVIEW Of FUNNCTION (XII)

- f:x→y such that f(x)=y is one-one and onto then we can define unique function g:y →x such that g(y)=x , where x ∊X , y ∊Y of y=f(x), here domain of g=range of f and range of g =domain of f . g is called inverse of f and denoted by f inverse g is also one-one and onto and inverse of g is f. so

$$(f^{-1}of)(x) = f^{-1}\big(f(x)\big) = x$$

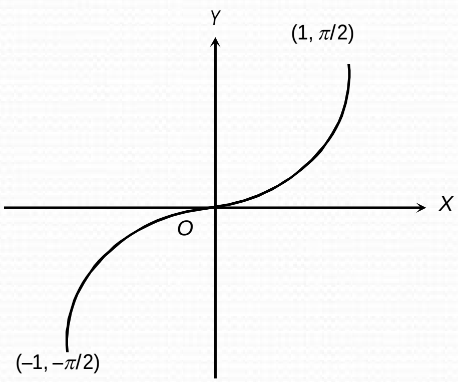
$$(fof^{-1})(y) = f\big(f^{-1}(y)\big) = f(x) = y$$

LINK WITH VIDEO ACTIVITY

https://www.youtube.com/watch?v=-E06UHenFxw

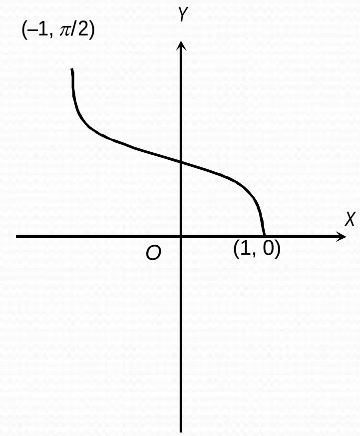
GRAPH OF INVERSE TRIGO FUNCTION

$f(x) = \sin^{-1} x$



$D_{f(x)} = [-1, 1]$ and $R_{f(x)} = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

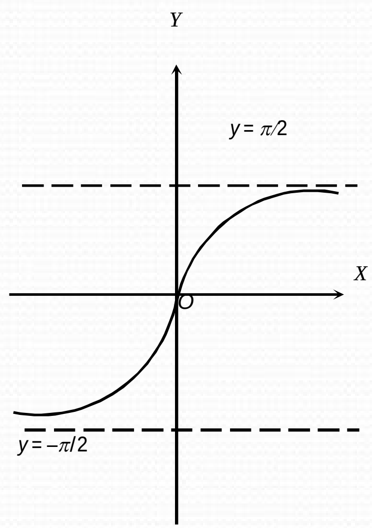
$f(x) = \cos^{-1} x$



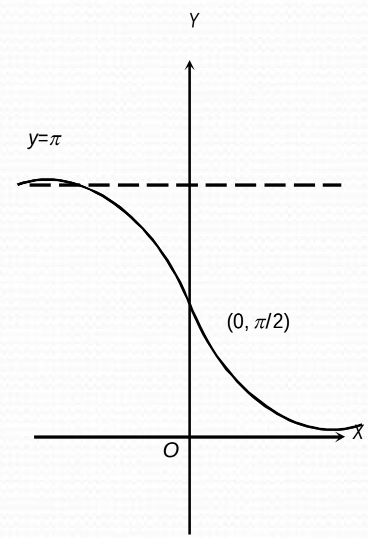
$D_{f(x)} = [-1, 1]$ & $R_{f(x)} = [0, \pi]$

GRAPH OF INVERSE TRIGO FUNCTION

$$f(x) = \tan^{-1} x$$



$$f(x) = \cot^{-1} x$$

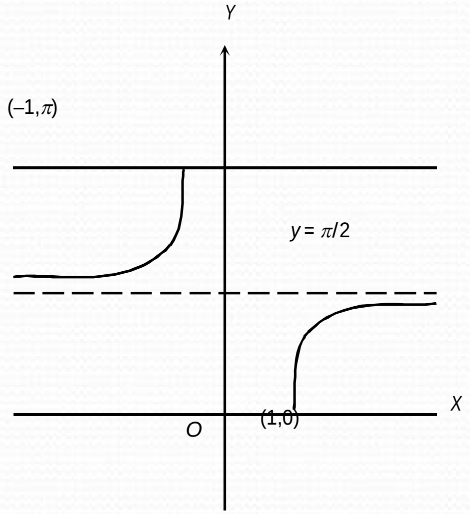


$$D_{f(x)} = R \quad and \quad R_{f(x)} = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

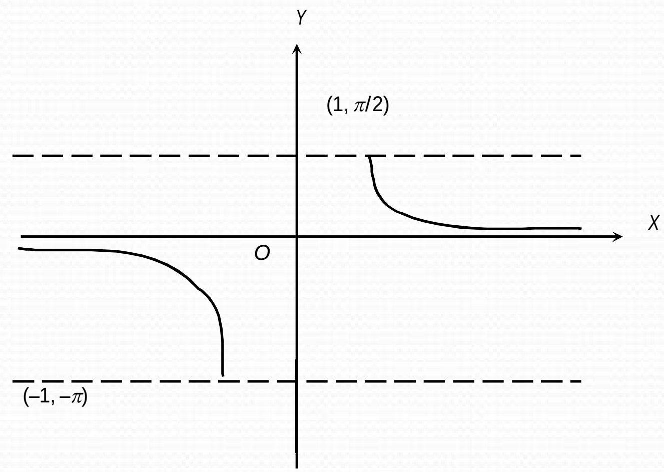
$$D_{f(x)} = R \quad and \quad R_{f(x)} = (0, \pi)$$

GRAPH OF INVERSE TRIGO FUNCTION

$f(x) = sec^{-1}x$



$f(x) = cosec^{-1}x$



$D_{f(x)} = R - (-1,1)$ and $R_{f(x)} = [0,\pi] - \{\frac{\pi}{2}\}$ $D_{f(x)} = R - (-1,1)$ and $R_{f(x)} = [-\frac{\pi}{2}, \frac{\pi}{2}] - \{0\}$

DOMAIN AND RANGE OF INVERSE TRIGONOMETRY FUNCTION

FUNCTION	DOMAIN	RANGE
$f(x) = sin^{-1}x$	[-1 ,1]	R
$f(x) = cos^{-1}x$	[-1 ,1]	R
$f(x) = tan^{-1}x$	R	$(-\frac{\pi}{2},\frac{\pi}{2})$
$f(x) = cot^{-1}x$	R	$(0,\pi)$
$f(x) = sec^{-1}x$	R-(-1 , 1)	$[0,\pi]-\{\frac{\pi}{2}\}$
$f(x) = cosec^{-1}x$	R-(-1 , 1)	$[-\frac{\pi}{2},\frac{\pi}{2}]$ $\{0\}$

PROPERTIES OF INVERSE TRIGO FUNCTION

SL NO	FUNCTION	CONDITION
1	$\sin^{-1}(\sin\theta) = \theta$	$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$
2	$\cos^{-1}(\cos\theta) = \theta$	$0 \le \theta \le \pi$
3	$\tan^{-1}(\tan\theta) = \theta$	$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$
4	$\cot^{-1}(\cot\theta) = \theta$	$0 < \theta < \pi$
5	$\sec^{-1}(\sec\theta) = \theta$	$0 \le \theta < \frac{\pi}{2} or \frac{\pi}{2} < \theta \le \pi$
6	$\mathrm{cosec}^{-1}(\mathrm{cosec}\,\theta) = \theta,$	$-\frac{\pi}{2} \le \theta < 0 or 0 < \theta \le \frac{\pi}{2}$

PROPERTIES OF INVERSE TRIGO FUNCTION

SL NO	FUNCTION	CONDITION
1	$\sin(\sin^{-1} x) = x$	[-1 , 1]
2	$\cos(\cos^{-1} x) = x$	[-1 , 1]
3	$\tan(\tan^{-1} x) = x$	R
4	$\cot(\cot^{-1} x) = x$	R
5	$\sec(\sec^{-1} x) = x$	R-(-1 , 1)
6	$\operatorname{cosec}(\operatorname{cosec}^{-1} x) = x$	R-(-1 , 1)

INSTRUCTION

$$1.\sin^{-1}(\sin 10) = 10 \quad , \textit{False}$$

$$2.\tan^{-1}(\tan 10) = 10 \quad , \textit{true}$$

PRINCIPAL VALUE CALCULATION

Principal values for $x \geq 0$	Principal values for $x < 0$
$0 \leq \sin^{-1} x \leq \frac{\pi}{2}$	$-\frac{\pi}{2} \leq \sin^{-1} x < 0$
$0 \leq \cos^{-1} x \leq \frac{\pi}{2}$	$\frac{\pi}{2} < \cos^{-1} x \leq \pi$
$0 \leq \tan^{-1} x < \frac{\pi}{2}$	$-\frac{\pi}{2} < \tan^{-1} x < 0$
$0 < \cot^{-1} x \leq \frac{\pi}{2}$	$\frac{\pi}{2} < \cot^{-1} x < \pi$
$0 \leq \sec^{-1} x < \frac{\pi}{2}$	$\frac{\pi}{2} < \sec^{-1} x \leq \pi$
$0 < \operatorname{cosec}^{-1} x \leq \frac{\pi}{2}$	$-\frac{\pi}{2} \leq \operatorname{cosec}^{-1} x < 0$

PROPERTIES OF INVERSE TRIGO FUNCTION

$$1.\ \sin^{-1}(-x) = -\sin^{-1} x,\ x \in [-1,1]$$

$$2.\ \cos^{-1}(-x) = \pi - \cos^{-1} x,\ x \in [-1,1]$$

$$3.\ \tan^{-1}(-x) = -\tan^{-1} x,\ x \in R$$

$$4.\ \cot^{-1}(-x) = \pi - \cot^{-1} x,\ x \in R$$

$$5.\ \sec^{-1}(-x) = \pi - \sec^{-1} x,\ x \in R - (-1,1)$$

$$6.\ \mathrm{cosec}^{-1}(-x) = -\mathrm{cosec}^{-1} x,\ x \in R - (-1,1)$$

EXAMPLES

$$1.\cos^{-1}\left(\cos\frac{7\pi}{6}\right)=?$$

$$Soln:\cos^{-1}\left(\cos\frac{7\pi}{6}\right)=\cos^{-1}\left\{\cos\left(\pi+\frac{\pi}{6}\right)\right\}$$

$$\cos^{-1}\left(-\cos\frac{\pi}{6}\right)=\pi-\cos^{-1}\cos\frac{\pi}{6}=\pi-\frac{\pi}{6}=\frac{5\pi}{6}$$

$$2.\sin^{-1}\left(-\frac{1}{2}\right)=?$$

$$Soln:\sin^{-1}\left(-\frac{1}{2}\right)=\sin^{-1}\sin(-30^{o})$$

$$=-\frac{\pi}{6}$$

EXAMPLES

$$1.\sin^{-1}(\sin 10) = ?$$

$$So\ln: 3\pi < 10 < 3\pi + \frac{\pi}{2} \Rightarrow 0 < 10 - 3\pi < \frac{\pi}{2}$$

$$\Rightarrow \frac{-\pi}{2} < 3\pi - 10 < 0$$

$$\Rightarrow \sin^{-1}\{\sin(3\pi - 10)\} = 3\pi - 10$$

FIND THE PRINCIPAL SOLUTION

1. $sin^{-1}(-\frac{1}{2})$

2. $cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

3. $cosec^{-1}(2)$

4. $tan^{-1}(-\sqrt{3})$

5. $cos^{-1}(-1/2)$

6. $tan^{-1}1 + cos^{-1}\left(-\frac{1}{2}\right) + sin^{-1}\left(-\frac{1}{2}\right)$

7. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$

8. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

PROPERTIES OF INVERSE TRIGO FUNCTION

$$1.\ \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2},\ x \in [-1,1]$$

$$2.\ \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2},\ x \in R$$

$$3.\ \sec^{-1}x + \text{cosec}^{-1}x = \frac{\pi}{2},\ x \in R-(-1,1)$$

INSTRUCTION

$$1.\sin^{-1}10+\cos^{-1}10=\frac{\pi}{2}, = False, because, x\notin[-1,1]$$

$$2.\tan^{-1}10+\cot^{-1}10=\frac{\pi}{2}, = True, Because, x\in R$$

CONVERSION PROPERTY

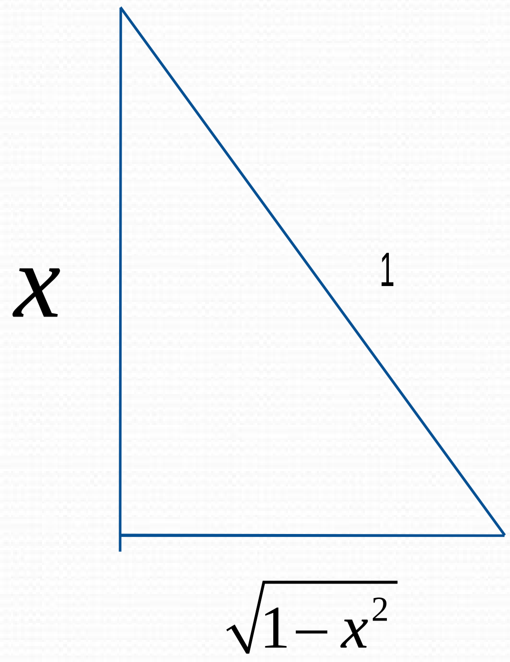
$$let \sin^{-1} x = y \Rightarrow x = \sin y \Rightarrow \operatorname{cosec} y = \left(\frac{1}{x}\right) \Rightarrow y = \operatorname{cosec}^{-1}\left(\frac{1}{x}\right)$$

$$\sin^{-1} x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{x}{\sqrt{1-x^2}} = \cot^{-1}\frac{\sqrt{1-x^2}}{x} = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right) = \operatorname{cosec}^{-1}\left(\frac{1}{x}\right)$$

$$\cos^{-1} x = \sin^{-1}\sqrt{1-x^2} = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) = \sec^{-1}\frac{1}{x} = \operatorname{cosec}^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right) = \cot^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

$$\tan^{-1} x = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right) = \cot^{-1}\left(\frac{1}{x}\right) = \sec^{-1}\sqrt{1+x^2} = \operatorname{cosec}^{-1}\left(\frac{\sqrt{1+x^2}}{x}\right)$$

EASY TO ANALYZE



$$\sin^{-1} x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{x}{\sqrt{1-x^2}} = \cot^{-1}\frac{\sqrt{1-x^2}}{x} = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right) = \operatorname{cosec}^{-1}\left(\frac{1}{x}\right)$$

CONVERSION PROPERTY

$$1.\ \sin^{-1}\left(\frac{1}{x}\right) = \operatorname{cosec}^{-1}x \quad , x \in (-\infty, 1] \cup [1, \infty)$$

$$2.\ \cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1}x, \quad x \in (-\infty, 1] \cup [1, \infty)$$

$$3.\ \tan^{-1}\left(\frac{1}{x}\right) = \begin{cases} \cot^{-1}x, & \text{for } x > 0 \\ -\pi + \cot^{-1}x, & \text{for } x < 0 \end{cases}$$

EXAMPLES

$$1.\ \tan^{-1}\frac{x}{\sqrt{a^2-x^2}}=?$$

$$so\ln : put, x = a\sin\theta$$

$$\tan^{-1}\frac{x}{\sqrt{a^2-x^2}}=\tan^{-1}\left(\frac{a\sin\theta}{a\cos\theta}\right)$$

$$=\tan^{-1}(\tan\theta)=\theta=\sin^{-1}\left(\frac{x}{a}\right)$$

$$2.\ \tan\left(\sec^{-1}\sqrt{1+x^2}\right)=?$$

$$so\ln : put, x=\tan\theta$$

$$\tan\left(\sec^{-1}\sqrt{1+x^2}\right)=\tan\left(\sec^{-1}\sqrt{1+\tan^2\theta}\right)$$

$$=\tan(\sec^{-1}\sec\theta)=\tan\theta=x$$

EXAMPLES

$$1.\ \sin[\cot^{-1}(\cos\ \tan^{-1} x)] = ?$$

$$Soln: \sin\left[\cot^{-1}\left(\cos\ \cos^{-1}\frac{1}{\sqrt{1+x^2}}\right)\right]$$

$$= \sin\left[\cot^{-1}\frac{1}{\sqrt{1+x^2}}\right] = \sin\left[\sin^{-1}\sqrt{\frac{1+x^2}{2+x^2}}\right]$$

$$= \sqrt{\frac{1+x^2}{2+x^2}}$$

SOLVE THE FOLLOWING

1. $\cos(\tan^{-1} x) = ?$

2. $\tan\left[\sec^{-1}\sqrt{1+x^2}\right] = ?$

3. $\tan^{-1}\left[\dfrac{\cos x}{1+\sin x}\right] = ?$

4. $\tan^{-1}\dfrac{1}{\sqrt{x^2-1}} = ?$

5. $\sec^2(\tan^{-1} 2) + \operatorname{cosec}^2(\cot^{-1} 3) = ?$

6. $\sin\cot^{-1}\tan\cos^{-1} x = ?$

7. $\sin^{-1}\dfrac{\sqrt{x}}{\sqrt{x+a}} = ?$

8. $if\ \sin\left(\sin^{-1}\dfrac{1}{5}+\cos^{-1}x\right) = 1,\ find x$

9. $if\ \dfrac{\pi}{2} \le x \le \dfrac{3\pi}{2},\quad then \sin^{-1}(\sin x) = ?$

10. $if\ \sin(\cot^{-1}(x+1) = \cos(\tan^{-1} x), then x = ?$

PROPERTIES

$$1.\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right) \quad , x>0, y>0, xy<1$$

$$\tan^{-1}x+\tan^{-1}y=\prod+\tan^{-1}\left(\frac{x+y}{1-xy}\right), x>0, y>0, xy>1$$

$$\tan^{-1}x+\tan^{-1}y=-\pi+\tan^{-1}\left(\frac{x+y}{1-xy}\right) \quad , x<0, y<0, xy>1$$

PROPERTIES

$$1. \tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right) \quad , x > 0, y < 0, xy > -1$$

$$\tan^{-1} x - \tan^{-1} y = \prod + \tan^{-1}\left(\frac{x-y}{1+xy}\right) \quad , x > 0, y < 0, xy < -1$$

$$\tan^{-1} x - \tan^{-1} y = -\prod + \tan^{-1}\left(\frac{x-y}{1+xy}\right) , x < 0, y > 0, xy < -1$$

SOLVE THE FOLLOWING

$$1.\ \tan\left[\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{2}{3}\right]=?$$

$$5.\ if\ \tan^{-1}\frac{x-1}{x+2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}\ then\ x=?$$

$$2.\ \tan^{-1}1+\tan^{-1}2+\tan^{-1}3=?$$

$$3.\ if\ \tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4}\ then\ x=?$$

$$4.\ if\ \tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\frac{\pi}{2},\ then\ x+y+z=?$$

PROPERTIES

$$1.\sin^{-1}x+\sin^{-1}y=\sin^{-1}\{x\sqrt{1-y^2}+y\sqrt{1-x^2}\}$$

$$-1\le x,y\le 1, x^2+y^2\le 1, xy<0, x^2+y^2>1$$

$$2.\sin^{-1}x-\sin^{-1}y=\sin^{-1}\{x\sqrt{1-y^2}-y\sqrt{1-x^2}\},$$

$$-1\le x,y\le 1, x^2+y^2\le 1, xy>0, x^2+y^2>1$$

PROPETRIES

$$1.\cos^{-1}x+\cos^{-1}y=\cos^{-1}\{xy-\sqrt{1-x^2}.\sqrt{1-y^2}\}$$

$$-1\leq x,y\leq 1, x+y\geq 0$$

$$2.\cos^{-1}x-\cos^{-1}y=\cos^{-1}\{xy+\sqrt{1-x^2}\sqrt{1-y^2}\},$$

$$-1\leq x,y\leq 1, x\leq y$$

EXAMPLES

$$1.\tan\left[\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{2}{3}\right]=?$$

$$So\ln:\tan\left[\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{2}{3}\right]$$

$$=\tan\left[\tan^{-1}\frac{\sqrt{\left(1-\frac{16}{25}\right)}}{\frac{4}{5}}+\tan^{-1}\frac{2}{3}\right]$$

$$=\tan\left[\tan^{-1}\left(\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}.\frac{2}{3}}\right)\right]=\tan.\tan^{-1}\frac{17}{6}=\frac{17}{6}$$

$$2.\tan^{-1}1+\tan^{-1}2+\tan^{-1}3=?$$

$$so\ln:\tan^{-1}1+\tan^{-1}2+\tan^{-1}3=?$$

$$=\tan^{-1}1+\pi+\tan^{-1}\left(\frac{5}{-5}\right)$$

$$=\tan^{-1}1+\pi-\tan^{-1}1=\pi$$

ANSWER THE FOLLOWING

$$1.\cot^{-1}\frac{3}{4}+\sin^{-1}\frac{5}{13}=\sin^{-1}?$$

$$5.\sin^{-1}\frac{3}{5}+\cos^{-1}\left(\frac{12}{13}\right)=\sin^{-1}C, then C=?$$

$$2.\cos^{-1}\frac{3}{5}-\sin^{-1}\frac{4}{5}=\cos^{-1}x, then x=?$$

$$3.\sin^{-1}\left(\frac{3}{5}\right)+\tan^{-1}\left(\frac{1}{7}\right)=\cos^{-1}?$$

$$4. if\ \sin^{-1}a+\sin^{-1}b+\sin^{-1}c=\pi,$$

$$then (a\sqrt{(1-a^2)}+b\sqrt{(1-b^2)}+c\sqrt{(1-c^2)})=?$$

PROPERTIES

$$1.\ 2\sin^{-1}x = \sin^{-1}(2x\sqrt{1-x^2}) \quad , -\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$$

$$2\sin^{-1}x = \pi - \sin^{-1}2x\sqrt{1-x^2} \quad , \frac{1}{\sqrt{2}} \le x \le 1$$

$$2\sin^{-1}x = -\pi - \sin^{-1}(2x\sqrt{1-x^2}) \quad , -1 \le x \le \frac{-1}{\sqrt{2}}$$

PROPERTIES

1. $2\cos^{-1}x = \cos^{-1}(2x^2 - 1)$, $0 \le x \le 1$

$2\cos^{-1}x = 2\pi - \cos^{-1}(2x^2 - 1)$ $-1 \le x \le 0$

PROPERTIES

$$1.\ 2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right) \quad , -1 < x \le 1$$

$$2\tan^{-1}x = \pi + \tan^{-1}\left(\frac{2x}{1-x^2}\right) \quad , x > 1$$

$$2\tan^{-1}x = -\pi + \tan^{-1}\left(\frac{2x}{1-x^2}\right) \quad , x < -1$$

ANSWER THE FOLLOWING

$$1.\ 2\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{2}=?$$

$$5.\ \cos^{-1}\frac{1}{2}+2\sin^{-1}\frac{1}{2}=?$$

$$2.\ 2\sin^{-1}\frac{3}{5}+\cos^{-1}\frac{24}{25}=?$$

$$6.\ \sin^{-1}\frac{4}{5}+2\tan^{-1}\frac{1}{3}=?$$

$$3.\ \cos\left[2\cos^{-1}\frac{1}{5}+\sin^{-1}\frac{1}{5}\right]=?$$

$$7.\ 4\sin^{-1}x+\cos^{-1}x=\pi, then x=?$$

$$4.\ 2\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{7}\right)=?$$

$$8.\ \sin\left\{\tan^{-1}\left(\frac{1-x^2}{2x}\right)+\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right\}=?$$

PROPERTIES

$$1.\ 3\sin^{-1}x = \sin^{-1}(3x-4x^3), \quad \frac{-1}{2} \le x \le \frac{1}{2}$$

$$3\sin^{-1}x = \pi - \sin^{-1}(3x-4x^3), \quad \frac{1}{2} < x \le 1$$

$$3\sin^{-1}x = -\pi - \sin^{-1}(3x-4x^3), \quad -1 \le x < -\frac{1}{2}$$

PROPERTIES

$$1.\ 3\cos^{-1}x = 2\pi - \cos^{-1}(4x^3 - 3x), \quad , -\frac{1}{2} \leq x \leq \frac{1}{2}$$

$$3\cos^{-1}x = 2\pi + \cos^{-1}(4x^3 - 3x), \quad , -1 \leq x \leq -\frac{1}{2}$$

PROPERTIES

$$1.\ 3\tan^{-1}x = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right), \quad \frac{-1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$$

$$3\tan^{-1}x = \pi + \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right), \quad x > \frac{1}{\sqrt{3}}$$

$$3\tan^{-1}x = -\pi + \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right), \quad x < -\frac{1}{\sqrt{3}}$$

EXAMPLES

$$1.\sum_{m=1}^{n}\tan^{-1}\left(\frac{2m}{m^4+m^2+2}\right)=?$$

$$Soln:\sum_{m=1}^{n}\tan^{-1}\left(\frac{2m}{1+(m^2+m+1)(m^2-m+1)}\right)$$

$$=\sum_{m=1}^{n}\tan^{-1}\left(\frac{(m^2+m+1)-(m^2-m+1)}{1+(m^2+m+1)(m^2-m+1)}\right)$$

$$=\sum_{m=1}^{n}[\tan^{-1}(m^2+m+1)-\tan^{-1}(m^2-m+1)]$$

$$=(\tan^{-1}3-\tan^{-1}1)+(\tan^{-1}7-\tan^{-1}3)+$$

$$(\tan^{-1}13-\tan^{-1}7)+......+[\tan^{-1}(n^2+n+1)\quad-\tan^{-1}(n^2-n+1)]$$

$$=\tan^{-1}(n^2+n+1)-\tan^{-1}1$$

$$=\tan^{-1}\left(\frac{n^2+n}{2+n^2+n}\right)$$

ANSWER THE FOLLOWING

1. $3sin^{-1}\left(\frac{1}{2}\right) + 3tan^{-1}\left(\frac{3}{5}\right) = cos^{-1}?$

2. $3cos^{-1}\left(\frac{1}{2}\right) + 3sin^{-1}\left(\frac{3}{5}\right) = tan^{-1}?$

3. $3cos^{-1}\left(\frac{1}{3}\right) + 3tan^{-1}\left(\frac{4}{5}\right) = cosec^{-1}?$

PROPERTIES

$$1.\ 2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right) \quad , -1 \le x \le 1$$

$$2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) \quad , 0 \le x < \infty$$

$$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right) \quad , -1 < x \le 1$$

ANSWER THE FOLLOWING

1. $if\ \sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\frac{\pi}{2}$, $x^2+y^2+z^2+2xyz=?$

2. $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$, $xyz=?$

3. $if\ \tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\frac{\pi}{2}$, $xyz=?$

4. $\cos^{-1}\sqrt{p}+\cos^{-1}\sqrt{1-p}+\cos^{-1}\sqrt{1-q}=\frac{3\pi}{4}$, $q=?$

5. $3\sin^{-1}\frac{2x}{1-x^2}-4\cos^{-1}\frac{1-x^2}{1+x^2}+2\tan^{-1}\frac{2x}{1-x^2}=\frac{\pi}{3}$, $x=?$

ANSWER THE FOLLOWING

1. $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right) = ?$

2. $\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{5} - \tan^{-1}\frac{8}{19} = ?$

3. $4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{70} + \tan^{-1}\frac{1}{99} = ?$

4. $If A \leq (\sin^{-1}x)^3 + (\cos^{-1}x)^3 \leq B, (A, B) = ?$

5. $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi, \frac{1}{xy} + \frac{1}{yz} + \frac{1}{zx} = ?$

ANSWER THE FOLLOWING

$$1.\cot^{-1}\left[\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right]=?$$

$$2.\cos^{-1}p+\cos^{-1}q+\cos^{-1}r=\pi,\ p^2+q^2+r^2+2pqr=?$$

$$3.\tan\left[\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right]+\tan\left[\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right]=?$$

$$4.\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]=?$$

$$5.2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x),\ x=?$$

ANSWER THE FOLLOWING

$$1. If \cos^{-1} x - \cos^{-1} \frac{y}{2} = \alpha, 4x^2 - 4xy\cos\alpha + y^2 = ?$$

$$2. \sum_{m=1}^{n} \tan^{-1}\left(\frac{2m}{m^4 + m^2 + 2}\right) = ?$$

$$3. \tan\left[\tan^{-1}\frac{d}{1 + a_1 a_2} + \tan^{-1}\frac{d}{1 + a_2 a_3} + ... + \tan^{-1}\frac{d}{1 + a_{n-1} a_n}\right] = ?$$

if, $a_1, a_2, a_3 \dots a_n$ is an A.P with common difference d

THANK YOU