**Class XII**

**Linear programming: Tutorial 1**

**Solving linear inequalities**

**Introduction:** These tutorials in mathematics are aimed at facilitating learning and understanding of the class 12 syllabus even when the school is not working.

We start with chapter 12 on '**Linear Programming'**.

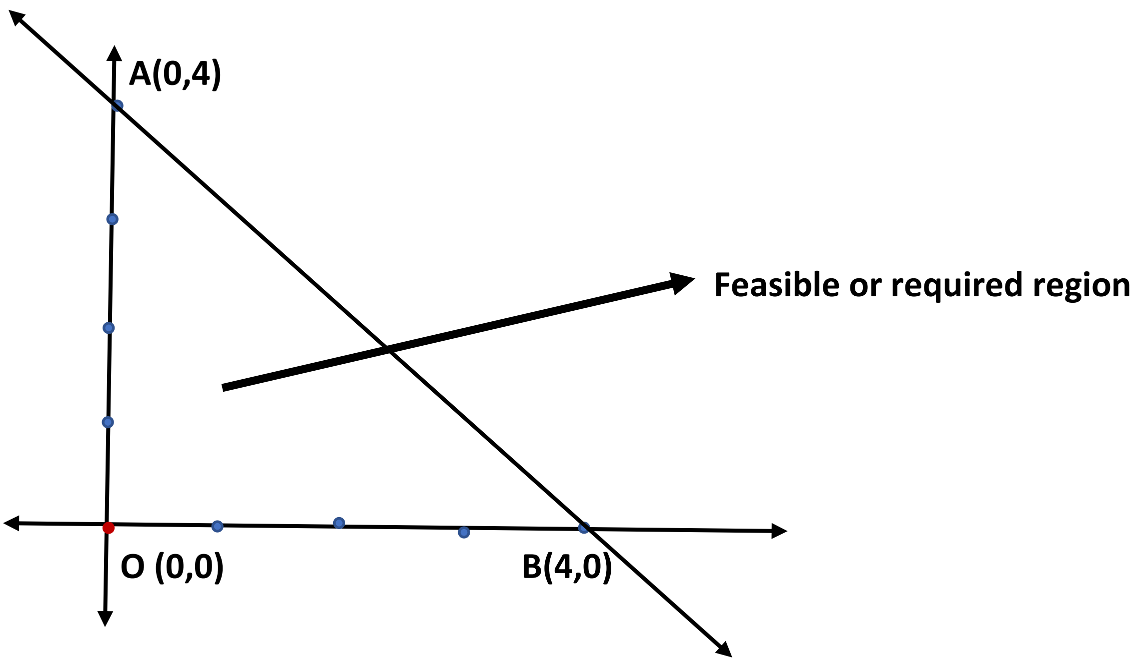
This short tutorial is a recall of the topic '**Linear Inequalities'** which you have already done in class 11.

I shall explain the graphical solution of linear inequalities with the help of examples.

Example 1: Solve graphically the linear inequality $\boldsymbol{x+y \leq 4, x \geq 0}$ & $\boldsymbol{y \geq 0}$

**Solution:** Since the values of **x and y** are **non-negative, the graph lies** in the **first quadrant.**

**Step 1:** Plot the graph of the linear equation **x + y = 4.**




It is a straight line passing through the points (0,4) and (4,0).

**Step 2:** the line **x + y = 4** divides the cartesian plane into two half planes.

**Step 3:** Choose any **one point** in **any one half plane** say **(0,0).**

**Step4:** Substitute (0,0) in the given inequality $\boldsymbol{0+\ 0 \leq 4\ is\ true}$

Thus, the region or **half plane containing the point (0,0)** is the required **feasible solution** of the given inequality.

Note that:

- If the inequality is **strict inequality**, that is, there is no equality sign, then the **line is not a part** of the feasible region

- If the inequality is not a strict inequality, that is, there is an equality sign, then the line is a part of the feasible region

Now, we shall study the simultaneous solution of two or more linear inequalities

**Example 2: $2x + y \geq 8$, $x + 2y \geq 10$**

**Solution:**

Given $2x + y \geq 8$

Putting value of x = 0 and y = 0 in equation one by one, we get value of

y = 8 and x = 4

The required points are (0, 8) and (4, 0)

Checking if the origin is included in the line`s graph (0, 0)

$0 \geq 8$, which is false

Hence the origin is not included in the solution area and the requires area would be the area to the right of line`s graph.

$x + 2y \geq 10$

Putting value of x = 0 and y = 0 in equation one by one, we get value of

y = 5 and x = 10

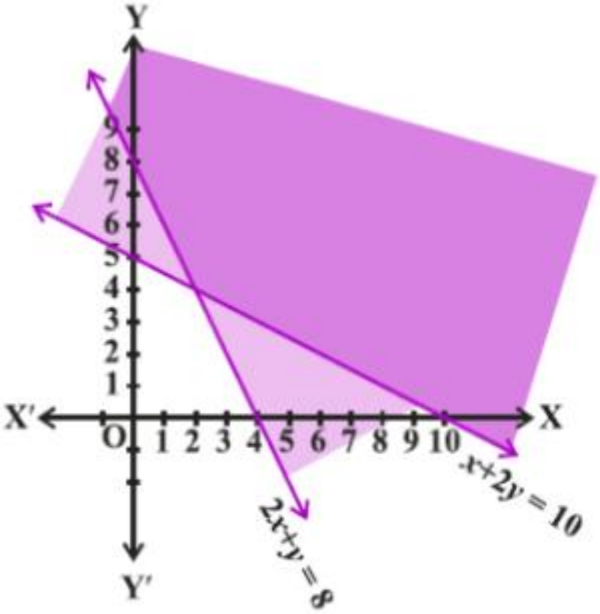
The required points are (0, 5) and (10, 0)

Checking for the origin (0, 0)

$0 \geq 10$ which is false,

Hence the origin would not lie in the required solution area. The required area would be to the left of the line graph.

The shaded area in the graph is the required solution of the given inequalities.




You can now go back to the content up loaded yesterday.

**Class XII**

**Linear Programming: Tutorial 2**

**Introduction:**

**Linear programming** is a branch of mathematical programming and is applied to solve **real life problems** by converting them into a **mathematical model**; that is a set of mathematical **equations/equalities** and **inequations/inequalities**.

The two key words are **'linear'** and **'programming'.**

**Linear** means all equations and inequations are **linear equations and inequations.**

**Programming** is the **process** of converting the **real life problem** into a **system of linear equations and inequations** and solving them mathematically.

We shall study the related terms with the help of an example.

A furniture dealer deals in two items – tables and chairs. He has ₹ 50,000/- to invest and has the storage space for at most 60 pieces of furniture. A table costs ₹ 2500 and a chair costs ₹ 500. He expects to earn a profit of ₹ 250 on selling one table and ₹ 75 on selling one chair. **How many tables and chairs** should he keep so that he earns maximum profit assuming that he can sell all items he keeps? What is his profit?

**Vocabulary:** Such problems in which we **maximize** or **minimize** profit or cost are called **optimization problems.**

We shall study three types of linear programming optimization problems:

- **Manufacturing problems**
- **Diet problems**
- **Transportation problems**

**Terms:**

**Assumptions:** The **variables** used to represent the **unknowns** in a problem are called **assumptions.**

**Constraints:** The linear **equations/inequalities** that is, **restrictions in a problem** are called **constraints.**

**Objective function:** The **linear function** to be **optimized (maximized or minimized)** is called **objective function.**

**Feasible region:** The set of **points** which satisfy all the **constraints** is called **feasible region.** It is either **bounded ( a polygon )** or an **unbounded** set of points.

**Optimal feasible solution:** The **set of point/s** which **maximize** or **minimize** the objective function is/are called **optimal solution/s.**

In a **bounded feasible region ( polygon )** the **optimal solution** is usually one of the **vertices** of the **polygon.**

**Solving a Linear Programming Problem graphically:**

Let us solve the problem given above.

**Assumptions:** Since, we are to find the number of tables and chairs, the assumptions are:

Let **x** be the **number of tables** and **y** be **number of chairs.**

**Constraints:** Obviously, number of tables and chairs cannot be negative so the first constraint is:

$$x \geq 0 \ and \ y \geq 0$$

The **money** he can **invest** is the next **constraint.**

Total cost of x tables and y chairs is 2500x + 500y

$$\therefore 2500x + 500y \leq 50,000 \ (investment \ constraint)$$

The **storage space** is the third constraint.

$$x + y \leq 60 \ (storage \ constraint)$$

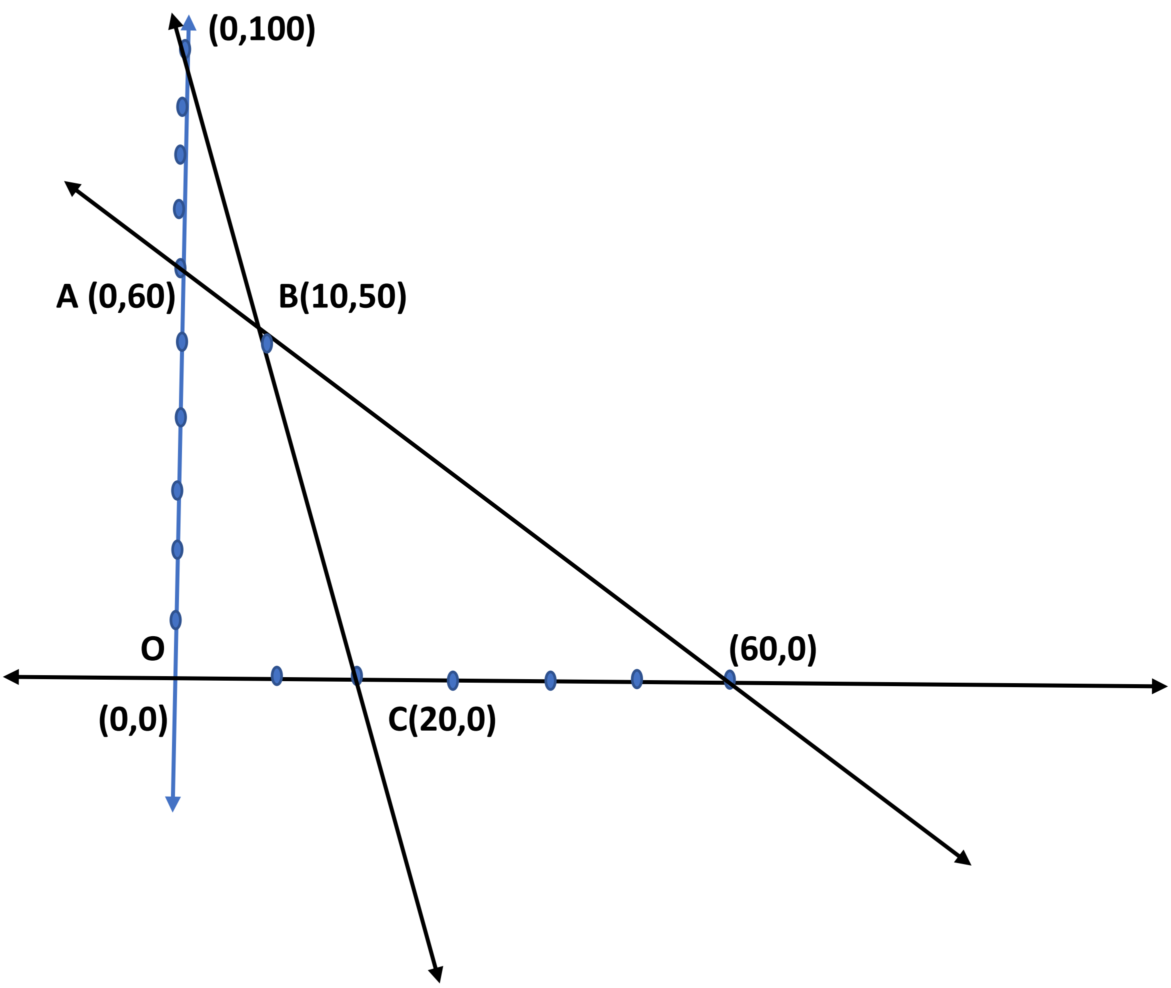
**Objective function:** The profit function has to be maximized

$$Z = 250x + 75y \ to \ be \ maximized$$

**The graphical solution:**

The **constraints** are $5x + y \leq 100 \ and \ x + y \leq 60$

The **graphs** of these **inequalities** are to be drawn in the **first quadrant.**



**Graphical solution: The feasible solution** is the quadrilateral **OABC.**

| **Corner points** | **Z = 500x+75y** |
|---|---|
| **O (0,0)** | **0** |
| **A (0,60)** | **4500** |
| **B(10,50)** | **6250 maximum** |
| **C(20,0)** | **5000** |

**Thus, the profit is maximum if the dealer deals in 10 tables and 50 chairs and the maximum profit is rupees 6250/-.**

Let us solve another problem.

An airline with a capacity of 200 passengers books two types of tickets – executive class and economy class. Atleast 20 seats have to be reserved for the executive class. But atleast four times the number of people travel by economy class than by the executive class. The profit made on each economy class ticket is 300 and executive class is 400. Formulate the Linear programming to maximize the profit made by the airline and solve the same to find the profit.

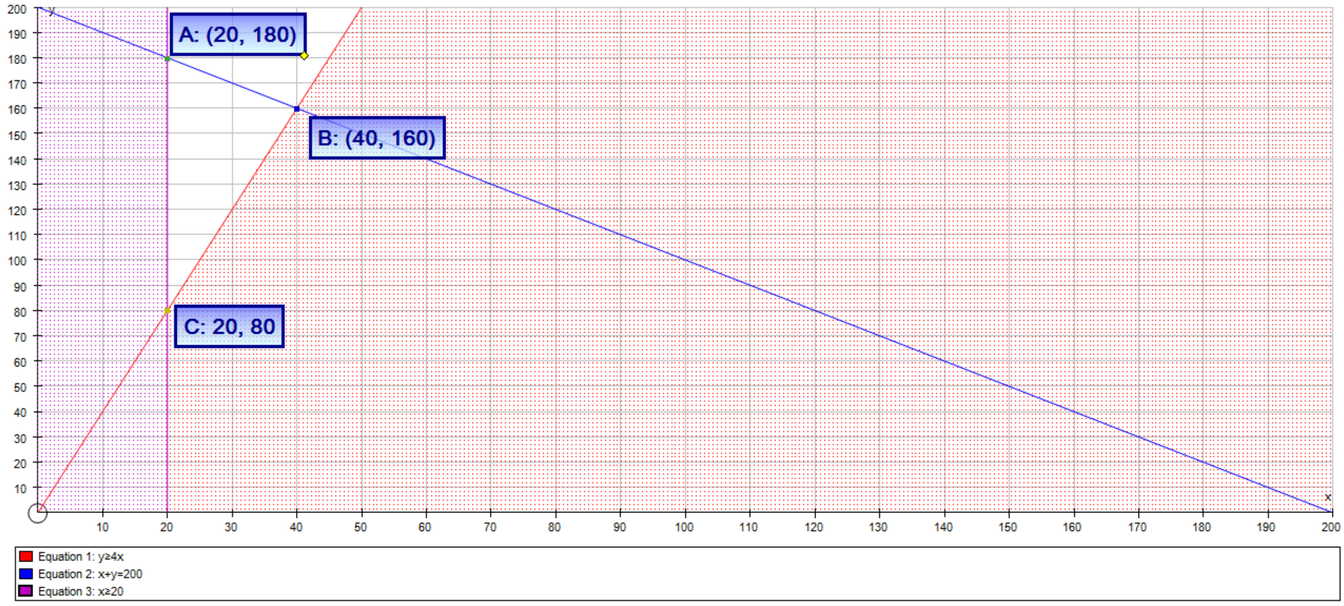
***Assumptions***: Let **x** be the number of executive class tickets and **y** the economy class tickets booked by the airline.

***Constraints***: $x \geq 0$, $y \geq 0$

$x + y \leq 200$

$x \geq 20$

$y \geq 4x$




***Objective function***: $z = 400x + 300y$ is to be maximized.

| Points | Z= 400x+300y |
|---|---|

| | |
|---|---|
| A(20,180) | 62,000 |
| B(40,160) | 64,000 maximum |
| C(20,80) | 32,000 |

Thus, the profit of the airline is maximum if 40 executive class tickets and 160 economy class tickets are sold.

**We shall take up problems on unbounded regions in the next tutorial class.**

**Try the NCERT exercises 12.1 and 12.2.**