DAV INSTITUTIONS ODISHA Zone-1

DAV PUBLIC SCHOOL

UNIT-VIII, BBSR-12

MATHEMATICS

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STD-XII
DETERMINANTS

INTRODUCTION

- The beginnings of matrices and determinants goes back to the second century BC although traces can be seen back to the fourth century BC. However it was not until near the end of the 17th Century that the ideas reappeared and development really got underway.
  It is not surprising that the beginnings of matrices and determinants should arise through the study of systems of linear equations. The Babylonians studied problems which lead to simultaneous linear equations and some of these are preserved in clay tablets which survive. For example a tablet dating from around 300 BC contains the following problem:-
  There are two fields whose total area is 1800 *square yards. One produces grain at the rate of 2/3 of a bushel per square yard while the other produces grain at the rate of* 1/2 *a bushel per square yard. If the total yield is* 1100 *bushels, what is the size of each field.*
- The Chinese, between 200 BC and 100 BC, came much closer to matrices than the Babylonians. Indeed it is fair to say that the text Nine Chapters on the Mathematical Art was written during the Han Dynasty gives the first known example of matrix methods. First a problem is set up which is similar to the Babylonian example given above:-

LEARNING OBJECTIVES

- *Students will be able to compute the determinant of a 2X2 , 3X3 amd higher order square matrices.*
- *Students learn about minors and cofactors of the elements of a determinant .*
- *Students will be able to interpret the effect of a determinant to find the area of a triangle with given vertices.*

SUB-TOPICS:-

- ❖Determinant of a Square Matrix
- ❖Minors and Cofactors
- ❖Properties of Determinants
- ❖Applications of Determinants
- ❖Area of a Triangle
- ❖Condition of Collinearity of Three Points

DETERMINANT

Every square matrix has associated with it a scalar called its determinant.

Given a matrix A, we use det(A) or |A| to designate its determinant.

We can also designate the determinant of matrix A by replacing the brackets by vertical straight lines. For example:

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \qquad \det(A) = \begin{vmatrix} 2 & 1 \\ 0 & 3 \end{vmatrix}$$

Definition 1: The determinant of a 1x1 matrix [a] is the scalar a.

Definition 2: The determinant of a 2x2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is the scalar ad-bc.

Sign System for Expansion of Determinant

Sign system for order 2 and order 3 are given by

$$\begin{vmatrix} + & - \\ - & + \end{vmatrix}, \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

Expansion of Determinants

If $A = [a_{ij}]$ is a square matrix of order 1, then $|A| = |a_{11}| = a_{11}$

If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is a square matrix of order 2, then

$$|A| = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = a_{11}a_{22} - a_{21}a_{12}$$

Example

Evaluate the determinant: $\begin{vmatrix} 4 & -3 \\ 2 & 5 \end{vmatrix}$

Solution: $\begin{vmatrix} 4 & -3 \\ 2 & 5 \end{vmatrix} = 4\times 5 - 2\times(-3) = 20 + 6 = 26$

Solution:

The determinant of a 3 × 3 matrix A, *where* $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ is a real number defined as

$$\det A = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - (a_{31}a_{22}a_{13} + a_{32}a_{23}a_{11} + a_{33}a_{21}a_{12}).$$

Solution:

If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ is a square matrix of order 3, then

$$|A| = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{23} \end{vmatrix} -$$

$$a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

Expanding along first row

$$= a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{31}a_{23}) + a_{13}(a_{21}a_{32} - a_{31}a_{22})$$

$$= (a_{11}a_{22}a_{33} + a_{12}a_{31}a_{23} + a_{13}a_{21}a_{32}) - (a_{11}a_{23}a_{32} + a_{12}a_{21}a_{33} + a_{13}a_{31}a_{22})$$

Example:

Evalueate the determinant : $\begin{vmatrix} 2 & 3 & -5 \\ 7 & 1 & -2 \\ 3 & 4 & 1 \end{vmatrix}$

$$\begin{vmatrix} 2 & 3 & -5 \\ 7 & 1 & -2 \\ 3 & 4 & 1 \end{vmatrix} = 2\begin{vmatrix} 1 & -2 \\ 4 & 1 \end{vmatrix} - 3\begin{vmatrix} 7 & -2 \\ -3 & 1 \end{vmatrix} + (-5)\begin{vmatrix} 7 & 1 \\ -3 & 4 \end{vmatrix}$$

[Expanding along first row]

$$= 2(1+8) - 3(7-6) - 5(28+3)$$

$$= 18 - 3 - 155$$

$$= -140$$

The Minor of an Element

- The determinant of each 3 × 3 matrix is called a **minor of the associated element.**
- The symbol M_{ij} *represents the minor when the ith row and jth column are eliminated.*

Element	Minor	Element	Minor
a_{11}	$M_{11} = \det\begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix}$	a_{22}	$M_{22} = \det\begin{bmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{bmatrix}$
a_{21}	$M_{21} = \det\begin{bmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{bmatrix}$	a_{23}	$M_{23} = \det\begin{bmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{bmatrix}$
a_{31}	$M_{31} = \det\begin{bmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{bmatrix}$	a_{33}	$M_{33} = \det\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

The Cofactor of an Element

Let M_{ij} *be the minor for element aij in an n × n matrix. The* ***cofactor of a_{ij}, written A_{ij}, is***

$$A_{ij} = (-1)^{i+j} \cdot M_{ij}.$$

- To find the determinant of a 3 × 3 or larger square matrix:

1. Choose any row or column,
2. Multiply the minor of each element in that row or column by a +1 or –1, depending on whether the sum of *i + j is even or odd,*
3. Then, multiply each cofactor by its corresponding element in the matrix and find the sum of these products. This sum is the determinant of the matrix.

Minors:

$$\text{If } A=\begin{bmatrix}-1 & 4\\ 2 & 3\end{bmatrix}, \text{then}$$

$$M_{11}=\text{Mior of } a_{11}=3, M_{12}=\text{Minor of } a_{12}=2$$

$$M_{21}=\text{Mior of } a_{21}=4, M_{22}=\text{Minor of } a_{22}=-1$$

$$\text{If } A=\begin{bmatrix}4 & 7 & 8\\ -9 & 0 & 0\\ 2 & 3 & 4\end{bmatrix}, \text{then}$$

$$M_{11}=\text{Mior of } a_{11}=\text{determinant of the order } 2\times 3 \text{ square sub-matrix is obtained by leaving first row and first column of } A=\begin{vmatrix}0 & 0\\ 3 & 4\end{vmatrix}=0$$

$$\text{Similarly } M_{23}=\text{minor of } a_{23}=\begin{vmatrix}4 & 7\\ 2 & 3\end{vmatrix}=12-14=-2$$

$$M_{32}=\text{minor of } a_{32}=\begin{vmatrix}4 & 8\\ -9 & 0\end{vmatrix}=0+72=72 \text{ etc.}$$

Cofactors

$$C_{ij} = \text{Cofactor of } a_{ij} \text{ in } A = (-1)^{i+j} M_{ij}, \text{ where } M_{ij} \text{ is minor of } a_{ij} \text{ in } A$$

Example

$$A = \begin{bmatrix} 4 & 7 & 8 \\ -9 & 0 & 0 \\ 2 & 3 & 4 \end{bmatrix}$$

$$C_{11} = \text{Cofactor of } a_{11} = (-1)^{1+1} M_{11} = (-1)^{1+1} \begin{vmatrix} 0 & 0 \\ 3 & 4 \end{vmatrix} = 0$$

$$C_{23} = \text{Cofactor of } a_{23} = (-1)^{2+3} M_{23} = -\begin{vmatrix} 4 & 7 \\ 2 & 3 \end{vmatrix} = 2$$

Value of Determinant in Terms of Minors and Cofactors

If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, then

$$|A| = \sum_{j=1}^{3} (-1)^{i+j} a_{ij} M_{ij} = \sum_{j=1}^{3} a_{ij} C_{ij}$$

$$= a_{i1}C_{i1} + a_{i2}C_{i2} + a_{i3}C_{i3}, \text{ for } i = 1 \text{ or } i = 2 \text{ or } i = 3$$

Dear children please go through this video first

https://youtu.be/hAh93-VHyu0

Properties of Determinant

1. The value of a determinant remains unchanged, if its rows and columns are interchanged.

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \text{ i.e. } |A| = |A'|$$

Example

$$\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$$

Expanding the determinant along first row,

$$\Delta = 2\begin{vmatrix} 0 & 4 \\ 5 & -7 \end{vmatrix} - (-3)\begin{vmatrix} 6 & 4 \\ 1 & -7 \end{vmatrix} + 5\begin{vmatrix} 6 & 0 \\ 1 & 5 \end{vmatrix}$$

$$= 2(0(-7) - 5(4) + 3(6(-7) - (1)(4)) + 5(6(5) - 1(0))$$

$$= 2(-20) + 3(-46) + 5(30) = -40 - 138 + 150 = -28$$

2.If any two rows (or columns) of a determinant are interchanged, then the value of the determinant is changed by minus sign.

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix} \text{ [Applying } R_2 \leftrightarrow R_1 \text{]}$$

Example

$$\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$$

Expanding the determinant along first row,

$$\Delta = 2\begin{vmatrix} 0 & 4 \\ 5 & -7 \end{vmatrix} - (-3)\begin{vmatrix} 6 & 4 \\ 1 & -7 \end{vmatrix} + 5\begin{vmatrix} 6 & 0 \\ 1 & 5 \end{vmatrix}$$

$$= 2(0(-7) - 5(4) + 3(6(-7) - (1)(4)) + 5(6(5) - 1(0))$$

$$= 2(-20) + 3(-46) + 5(30) = -40 - 138 + 150 = -28$$

Properties:

3. If all the elements of a row (or column) is multiplied by a non-zero number k, then the value of the new determinant is k times the value of the original determinant.

$$\begin{vmatrix} ka_1 & kb_1 & kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = k\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Which also implies

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \frac{1}{m}\begin{vmatrix} ma_1 & mb_1 & mc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Example

$$\text{Evaluate}\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix} \qquad \Delta = \begin{vmatrix} 6(17) & 6(3) & 6(6) \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$$

$$So, \Delta = \begin{vmatrix} 17 & 3 & 6 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix} = 6\times(0) = 0$$

Properties:

4. If each element of any row (or column) consists of two or more terms, then the determinant can be expressed as the sum of two or more determinants.

$$\begin{vmatrix} a_1+x & b_1 & c_1 \\ a_2+y & b_2 & c_2 \\ a_3+z & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} x & b_1 & c_1 \\ y & b_2 & c_2 \\ z & b_3 & c_3 \end{vmatrix}$$

Example

show that $\begin{vmatrix} a & b & c \\ a+2x & b+2x & c+2z \\ x & y & z \end{vmatrix} = 0$

Solving L.H.S $\begin{vmatrix} a & b & c \\ a+2x & b+2x & c+2z \\ x & y & z \end{vmatrix}$ expressing elements fo 2nd row as sum of two elements

$$= \begin{vmatrix} \mathrm{a} & \mathrm{b} & \mathrm{c} \\ \mathrm{a} & \mathrm{b} & \mathrm{c} \\ \mathrm{x} & \mathrm{y} & \mathrm{z} \end{vmatrix} + \begin{vmatrix} a & b & c \\ 2x & 2y & 2z \\ x & y & z \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ x & y & z \\ x & y & z \end{vmatrix}$$

R_2 and R_3 are identical

$= 2 \times 0 = 0$

5. The value of a determinant is unchanged, if any row (or column) is multiplied by a number and then added to any other row (or column).

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1+mb_1-nc_1 & b_1 & c_1 \\ a_2+mb_2-nc_2 & b_2 & c_2 \\ a_3+mb_3-nc_3 & b_3 & c_3 \end{vmatrix}$$

Properties:

6. If any two rows (or columns) of a determinant are identical, then its value is zero.

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \end{vmatrix} = 0$$

Example

$$\Delta = \begin{vmatrix} 3 & 2 & 3 \\ 2 & 2 & 3 \\ 3 & 2 & 3 \end{vmatrix}$$

first and third row are identical, hence we apply property 3

$$\Delta = 3\begin{vmatrix} 2 & 3 \\ 3 & 3 \end{vmatrix} - 2\begin{vmatrix} 2 & 3 \\ 3 & 3 \end{vmatrix} + 3\begin{vmatrix} 2 & 2 \\ 3 & 2 \end{vmatrix}$$

$$= 3((3) - 2(3)) - 2(2(3) - 3(3)) + 3(2(2) - 3(2))$$

$$= 3(6 - 6) - 2(6 - 9) + 3(4 - 6) = 0 + 6 - 6 = 0$$

7. If each element of a row (or column) of a determinant is zero, then its value is zero.

$$\begin{vmatrix} 0 & 0 & 0 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

8. Let $|A| = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ be a diagonalmatrix, then $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} = abc$

Example-1

Find the value of the following determinants

$$(i)\begin{vmatrix}42 & 1 & 6\\ 28 & 7 & 4\\ 14 & 3 & 2\end{vmatrix} \qquad (ii)\begin{vmatrix}6 & -3 & 2\\ 2 & -1 & 2\\ -10 & 5 & 2\end{vmatrix}$$

Solution:

$$(i)\begin{vmatrix}42 & 1 & 6\\ 28 & 7 & 4\\ 14 & 3 & 2\end{vmatrix} = \begin{vmatrix}6\times 7 & 1 & 6\\ 4\times 7 & 7 & 4\\ 2\times 7 & 3 & 2\end{vmatrix} = 7\begin{vmatrix}6 & 1 & 6\\ 4 & 7 & 4\\ 2 & 3 & 2\end{vmatrix} \quad [\text{Taking out 7 common from } C_1]$$

$$= 7\times 0 \qquad [\because C_1 \text{ and } C_3 \text{ are identical}]$$

$$= 0$$

$$(ii)\begin{vmatrix}6 & -3 & 2\\ 2 & -1 & 2\\ -10 & 5 & 2\end{vmatrix} = \begin{vmatrix}-3\times(-2) & -3 & 2\\ -1\times(-2) & -1 & 2\\ 5\times(-2) & 5 & 2\end{vmatrix}$$

$$= (-2)\begin{vmatrix}-3 & -3 & 2\\ -1 & -1 & 2\\ 5 & 5 & 2\end{vmatrix} \qquad [\text{Taking out -2 common from } C_1]$$

$$= (-2)\times 0 \qquad [\because C_1 \text{ and } C_2 \text{ are identidal}]$$

$$= 0$$

Example-2

Evalueate the determinant $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$

Solution:

$$\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = \begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix} \qquad [\text{Applying } C_2 \rightarrow C_2 + C_3]$$

$$= (a+b+c)\begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix} \qquad [\text{Taking } (a+b+c) \text{ common from } C_3]$$

$$= (a+b+c) \times 0 \qquad [\because C_1 \text{ and } C_3 \text{ are identical}]$$

$$= 0$$

Example - 3

Evaluate the determinant: $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix}$

Solution:

We have $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix}$

$$= \begin{vmatrix} (a-b) & b-c & c \\ (a-b)(a+b) & (b-c)(b+c) & c^2 \\ -c(a-b) & -a(b-c) & ab \end{vmatrix} \quad [\text{Applying } C_1 \to C_1 - C_2 \text{ and } C_2 \to C_2 - C_3]$$

$$= (a-b)(b-c)\begin{vmatrix} 1 & 1 & c \\ a+b & b+c & c^2 \\ -c & -a & ab \end{vmatrix} \quad \begin{bmatrix}\text{Taking } (a-b) \text{ and } (b-c) \text{ common} \\ \text{from } C_1 \text{ and } C_2 \text{ respectively}\end{bmatrix}$$

$$= (a-b)(b-c)\begin{vmatrix} 0 & 1 & c \\ -(c-a) & b+c & c^2 \\ -(c-a) & -a & ab \end{vmatrix} \quad [\text{Applying } C_1 \to C_1 - C_2]$$

$$= -(a-b)(b-c)(c-a)\begin{vmatrix} 0 & 1 & c \\ 1 & b+c & c^2 \\ 1 & -a & ab \end{vmatrix}$$

$$= -(a-b)(b-c)(c-a)\begin{vmatrix} 0 & 1 & c \\ 0 & a+b+c & c^2-ab \\ 1 & -a & ab \end{vmatrix} \quad [\text{Applying } R_2 \to R_2 - R_3]$$

Now expanding along C_1, we get

$(a-b)(b-c)(c-a)[-(c^2 - ab - ac - bc - c^2)]$

$= (a-b)(b-c)(c-a)(ab + bc + ac)$

Example-4

Without expanding the determinant,

prove that $\begin{vmatrix} 3x+y & 2x & x \\ 4x+3y & 3x & 3x \\ 5x+6y & 4x & 6x \end{vmatrix} = x^3$

Solution :

$$L.H.S = \begin{vmatrix} 3x+y & 2x & x \\ 4x+3y & 3x & 3x \\ 5x+6y & 4x & 6x \end{vmatrix} = \begin{vmatrix} 3x & 2x & x \\ 4x & 3x & 3x \\ 5x & 4x & 6x \end{vmatrix} + \begin{vmatrix} y & 2x & x \\ 3y & 3x & 3x \\ 6y & 4x & 6x \end{vmatrix}$$

$$= x^3 \begin{vmatrix} 3 & 2 & 1 \\ 4 & 3 & 3 \\ 5 & 4 & 6 \end{vmatrix} + x^2y \begin{vmatrix} 1 & 2 & 1 \\ 3 & 3 & 3 \\ 6 & 4 & 6 \end{vmatrix}$$

$$= x^3 \begin{vmatrix} 3 & 2 & 1 \\ 4 & 3 & 3 \\ 5 & 4 & 6 \end{vmatrix} + x^2y \times 0 \quad [\because C_1 \text{ and } C_3 \text{ are identical in II determinant}]$$

$$= x^3 \begin{vmatrix} 3 & 2 & 1 \\ 4 & 3 & 3 \\ 5 & 4 & 6 \end{vmatrix}$$

$$= x^3 \begin{vmatrix} 1 & 2 & 1 \\ 1 & 3 & 3 \\ 1 & 4 & 6 \end{vmatrix} \quad [\text{Applying } C_1 \to C_1 - C_2]$$

$$= x^3 \begin{vmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 3 \end{vmatrix} \quad [\text{Applying } R_2 \to R_2 - R_1 \text{ and } R_3 \to R_3 - R_2]$$

$$= x^3 \times (3-2) \quad [\text{Expanding along } C_1]$$

$$= x^3 = R.H.S.$$

Activity

Area of the triangle $\Delta = \frac{1}{2} \times base \times height$

$= \frac{1}{2} \times 10 \times 6$

$= 30$ sq. units

Vertices are A (4,9), B (1,3), C(11,3)

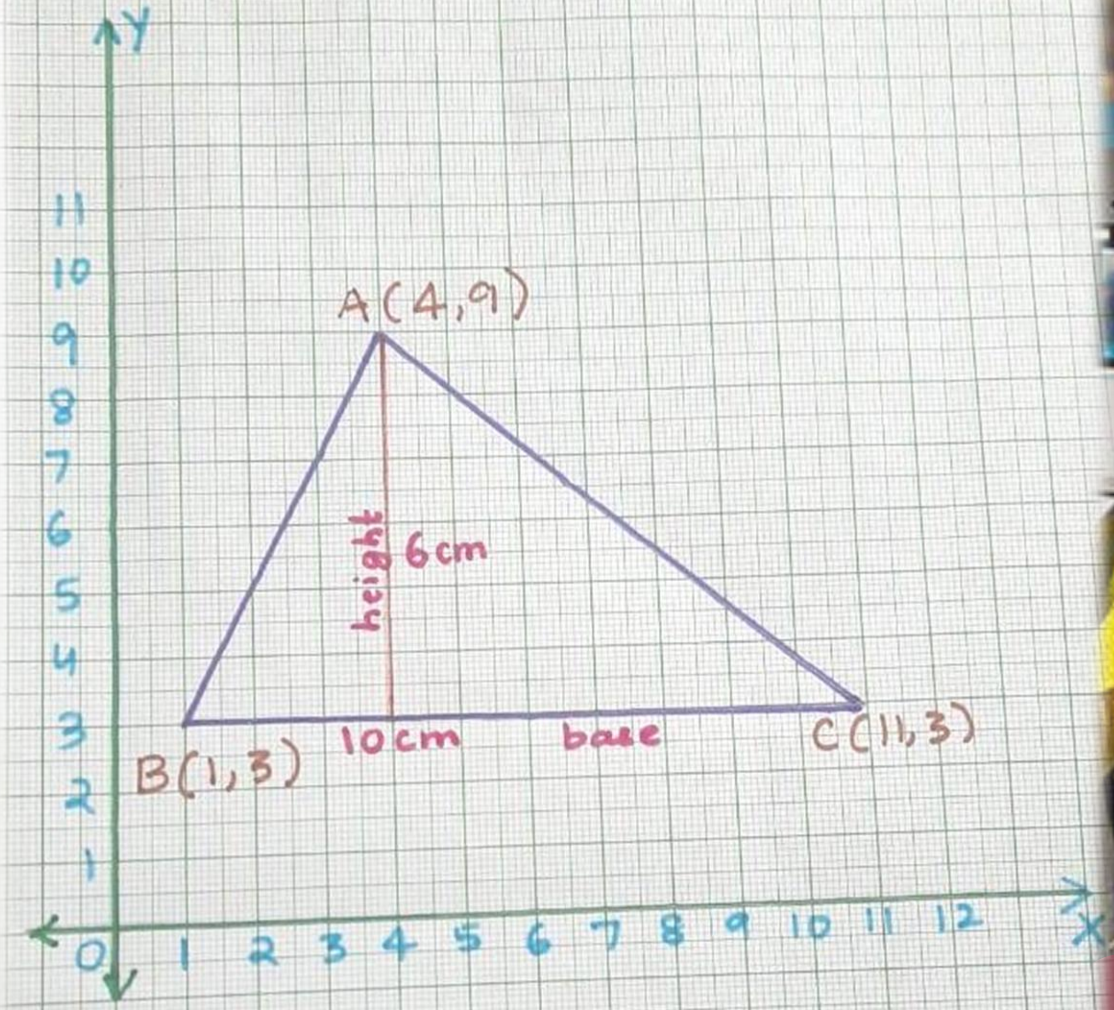
Area of the triangle

$$\Delta = \frac{1}{2}\begin{vmatrix} 4 & 9 & 1 \\ 1 & 3 & 1 \\ 11 & 3 & 1 \end{vmatrix}$$

$= \frac{1}{2}\left[4(3-3) - 9(1-11) + 1(3-33)\right]$

$= \frac{1}{2}(90 - 30)$

$= 30$ sq units



Applications of Determinants

(Area of a Triangle)

- The area of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by the expression

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$

Example:

Find the area of a triangle whose vertices are (-1,8),(-2,-3) and (3,2).

Solution:

$$\text{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -1 & 8 & 1 \\ 2 & -3 & 1 \\ 3 & 2 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[-1(-3-2)-8(-2-3)+1(-4+9)]$$

$$= \frac{1}{2}[5+40+5] = 25\,\text{sq.units}$$

Condition of Collinearity of Three Points

- If are three points, then A, B, C are collinear A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3)

$$\Leftrightarrow \text{Area of triangle } ABC = 0$$

$$\Leftrightarrow \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0 \Leftrightarrow \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

DETERMINANTS

CONCEPT MAP

Class XII

DETERMINANTS

Corresponding to every square matrix A, there exists a number called the determinant of A and denoted by $|A|$.

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$. Then,

$$|A| = a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

Properties

- The value of a determinant remains unaltered if its rows and columns are interchanged.
- If two rows (or columns) of a determinant are interchanged, the value of the determinant is multiplied by -1.
- If any two rows (or columns) of a determinant are identical, then the value of determinant is zero.
- If the elements of a row (or column) of a determinant are multiplied by any scalar, then the value of the new determinant is equal to same scalar times the value of the original determinant.
- If each element of any row (or column) of a determinant is the sum of two numbers, then the determinant is expressible as the sum of two determinants of the same order.

Note : (i) If $|A| = 0$, then the matrix is singular.
(ii) If $|A| \neq 0$, then the matrix is non-singular.

Minors and Cofactors

- For any matrix $A = [a_{ij}]_{n\times n}$, if we leave the row and the column of the element a_{ij}, then the value of determinant thus obtained is called the minor of a_{ij} and it is denoted by M_{ij}.
- The minor M_{ij} multiplied by $(-1)^{i+j}$ is called the cofactor of the element a_{ij} and denoted by A_{ij}.

$\therefore\ A_{ij} = (-1)^{i+j} M_{ij}$

Adjoint of a Matrix

Let $B = [A_{ij}]$ be the matrix of cofactors of matrix $A = [a_{ij}]$. Then the transpose of B is called the adjoint of matrix A.

Properties of adj (A)

- $A(\text{adj } A) = (\text{adj } A)\, A = |A|\, I_n$
- $\text{adj } (AB) = (\text{adj } B) \cdot (\text{adj } A)$
- $|\text{adj } A| = |A|^{n-1}$, where n is the order of A.
- $\text{adj } (\text{adj } A) = |A|^{n-2} A$
  $\Rightarrow |\text{adj } (\text{adj } A)| = |A|^{(n-1)^2}$

Inverse of a Matrix

For any square matrix A, inverse of A is defined as $A^{-1} = \frac{1}{|A|}(\text{adj } A)$, $|A| \neq 0$

Properties

- $(A^{-1})^{-1} = A$
- $(A')^{-1} = (A^{-1})'$
- $(AB)^{-1} = B^{-1}A^{-1}$
- $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

Area of a Triangle

Let ABC be a triangle with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$, then area of ΔABC is

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Solution of System of Linear Equations

Let $AX = B$ be the given system of equations :

- If $|A| \neq 0$, the system is consistent and has one unique solution.
- If $|A| = 0$ and $(\text{adj } A)B \neq O$, then the system is inconsistent and hence it has no solution.
- If $|A| = 0$ and $(\text{adj } A)B = O$, then the system may be either consistent or inconsistent according as the system has either infinitely many solutions or no solution.

Thank You.......