DAV INSTITUTIONS ODISHAZONE 1

MATRIX

DEPT. OF MATHEMATICS

DAV Public School
Pokhariput,
Bhubaneswar, Odisha

LEARNING OBJECTIVES:-

- Cost estimation, Sale projection and factory problems can be solved by using matrix.
- Expressing in vector form
- Expressing day to day life problems in matrix form
- Matrix notation and operations are used in electronic spreadsheet, advanced statistics.
- Expressing simultaneous linear equations in matrix form.

Defination of matrix:-

- A matrix is an ordered rectangular array of numbers that represent some data (Plural = matrices)
- A matrix on its own has no value – it is just a representation of data
- Could be data associated with manufactured quantity in a factory, speed of a rocket etc
- Forms the basis of computer programming
- A matrix is used in solving equations that represent business problems

Types of matrix :-

- **Row matrix:** it having only one row Ex $\begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$
- **Column matrix:** it having only one column Ex $\begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$
- **Zero matrix:** A matrix is called a zero matrix if all the entries are 0 Ex

  $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

- **Square matrix:** if number of rows is equal to number of columns $\begin{bmatrix} 5 & 7 \\ -2 & 5 \end{bmatrix}$
- **Note:** if number of rows = no of number columns =n, is called square matrix of order n or order n
- order 2 $\begin{bmatrix} 5 & 7 \\ -2 & 5 \end{bmatrix}$ order 3 $\begin{bmatrix} 5 & 7 & 8 \\ 6 & 4 & 8 \\ 1 & 7 & 0 \end{bmatrix}$

Types of matrix :-

- **Diagonal matrix:** A square matrix is called diagonal matrix, if all of its non-diagonal elements are zero.
- EXAMPLE

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- **Scalar matrix:** A square matrix is called scalar matrix if diagonal elements are same and other are "0"
- EXAMPLE

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

- **Identity/ unit matrix :** A square matrix is identity if diagonal entries are 1 and other are 0.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

REPRESENTATION OF MATRIX

$$A = [a_{ij}]_{MxN}$$

a_{ij} = element in row 'i' and column 'j', where 'a' is an element in the matrix

Eg: a_{23} = element in 2nd row and 3rd column = 9

Examples of Matrices

$$\begin{bmatrix} 2 & 4 \\ 5 & 7 \end{bmatrix}$$

This is an example of a 2 x 2 matrix

What is a_{12}

$$\begin{bmatrix} 2 & 3 & 6 \\ 72 & 3 & 9 \end{bmatrix}$$

What is the dimension or der of this Matrix?

What is a_{12}

$$\begin{bmatrix} 7 & 9 & 11 & 5 \\ 9 & 0 & 3 & 6 \end{bmatrix}$$

What is the dimension or order of this Matrix?

What is a_{12} ?

Addition /subtraction: when two matrices of same order are added/ subtracted, their corresponding entries are added/subtracted

Addition operation on Matrices

$$\underset{A}{\begin{bmatrix} 2 & 45 & 72 \\ 6 & 3 & 0 \\ 7 & 9 & 10 \end{bmatrix}} + \underset{B}{\begin{bmatrix} 40 & 7 & 9 \\ 6 & 1 & 2 \\ 7 & 2 & 8 \end{bmatrix}}$$

$$= \begin{bmatrix} (2+40) & (45+7) & (72+9) \\ (6+6) & (3+1) & (0+2) \\ (7+7) & (9+2) & (10+8) \end{bmatrix} = \begin{bmatrix} 47 & 52 & 81 \\ 12 & 4 & 2 \\ 14 & 11 & 18 \end{bmatrix}$$

Only Matrices of the same order(comparable) can be added!!

Rule 1: A + B =B + A

Question Set 1

1. Add the following matrices:

$$\begin{bmatrix} 32 & 4 & 60 \\ 29 & 2 & 4 \\ 21 & 65 & 7 \end{bmatrix} + \begin{bmatrix} 22 & 5 & 8 \\ 10 & 8 & 12 \\ 9 & 7 & 2 \end{bmatrix}$$

2. Subtract the following matrices:

$$\begin{bmatrix} 18 & 26 & 12 \\ 10 & 11 & 12 \\ 8 & 10 & 16 \end{bmatrix} - \begin{bmatrix} 7 & 2 & 15 \\ 13 & 3 & 5 \\ 5 & 8 & 9 \end{bmatrix}$$

Multiplication of a matrix by a scalar

If K is any number and A is a given matrix,

Then KA is the matrix obtained by multiplying each element of A by K.

K is called 'Scalar'. Eg: if K = 2

$$A = \begin{bmatrix} 2 & 4 & 5 \\ 1 & 3 & 2 \\ 2 & 5 & 1 \end{bmatrix} \quad KA = \begin{bmatrix} 4 & 8 & 10 \\ 2 & 6 & 4 \\ 4 & 10 & 2 \end{bmatrix}$$

MULTIPLICATION OF MATRICES

- The product AB of two matrices A and B is defined, if the number of columns of A is equal to the number of B.
- If AB is defined then BA need not be defined . In particular both A and B are square matrices of same order then AB and BA are defined.
- In general AB≠BA
- Observation : Two non zero matrices multiplication is zero matrix

Ex: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times2} \times \begin{bmatrix} -1 & 2 \\ 0 & 1 \\ 2 & 1 \end{bmatrix}_{3\times2}$ here multiplication is not possible.

Ex: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times2} \begin{bmatrix} 2 \\ 1 \end{bmatrix}_{2\times1}$ here multiplication possible, order of new matrix is 2×1.

If order of first matrix A is m$\times n$ and that of B is n$\times p$, then A.B is possible of order m$\times p$.

Multiplication of Matrices - 2

A

$$\begin{bmatrix} \boxed{2 \quad 3 \quad 1} \\ \boxed{4 \quad 3 \quad 2} \end{bmatrix} \times \begin{bmatrix} \boxed{\begin{matrix} 4 \\ 1 \\ 5 \end{matrix}} & \boxed{\begin{matrix} 2 \\ 0 \\ 2 \end{matrix}} \end{bmatrix}$$

2 x 3 matrix

B

3 x 2 matrix

$$= \begin{bmatrix} (2\times4+3\times1+1\times5) & (2\times2+3\times0+1\times2) \\ (4\times4+3\times1+2\times5) & (4\times2+3\times0+2\times2) \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 13 \\ 29 & 15 \end{bmatrix}$$

A x B

2 x 2 matrix

MLTIPLICATION OF MATRIX

Procedure for multiplication:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} =$$

$$\begin{bmatrix} a_{11}.b_{11} + a_{12}\ b_{21} + a_{13}\ b_{31} & a_{11}.b_{12} + a_{12}\ b_{22} + a_{13}\ b_{32} & a_{11}.b_{13} + a_{12}\ b_{23} + a_{13}\ b_{33} \\ a_{21}.b_{11} + a_{22}\ b_{21} + a_{23}\ b_{31} & a_{21}.b_{12} + a_{22}\ b_{22} + a_{23}\ b_{32} & a_{21}.b_{13} + a_{22}\ b_{23} + a_{23}\ b_{33} \\ a_{31}.b_{11} + a_{32}\ b_{21} + a_{33}\ b_{31} & a_{31}.b_{12} + a_{32}\ b_{22} + a_{33}\ b_{32} & a_{31}.b_{13} + a_{32}\ b_{23} + a_{33}\ b_{33} \end{bmatrix}$$

EX: $\begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix} \times \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1.4 + 2.-1 & 1.0 + 2.2 \\ -1.4 + 0.-1 & -1.0 + 0.2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ -4 & 0 \end{bmatrix}$

EX: $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1.1 + 2.3 \\ 1.1 + 1.3 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$

Multiplication of Matrices: - 1

$$\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 2 \\ 2 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 2 & 1 & 3 \\ 4 & 5 & 2 \\ 6 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} (1\times2 + 3\times4+5\times6) & (1\times1+ 3\times5+5\times2) & (1\times3+3\times2+5\times3) \\ (2\times2 + 4\times4+2\times6) & (2\times1+ 4\times5+2\times2) & (2\times3+4\times2+2\times3) \\ (2\times2 + 5\times4+6\times6) & (2\times1+ 5\times5+6\times2) & (2\times3+5\times2+6\times3) \end{bmatrix}$$

$$= \begin{bmatrix} 25 & 26 & 24 \\ 32 & 26 & 20 \\ 58 & 39 & 34 \end{bmatrix}$$

Multiplication of Matrices - 3

B

3 x 2 matrix

$$\begin{bmatrix} 4 & 2 \\ 1 & 0 \\ 5 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 3 & 1 \\ 4 & 3 & 2 \end{bmatrix}$$

A

2 x 3 matrix

$$= \begin{bmatrix} (4x2+2x4) & (4x3+2x3) & (4x1+2x2) \\ (1x2+0x4 & (1x3+0x3) & (1x1+0x2) \\ (5x2+2x4) & (5x3+2x3) & (5x1+2x2) \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 18 & 8 \\ 2 & 3 & 1 \\ 18 & 21 & 9 \end{bmatrix}$$

B xA

3 x 3 matrix

Rule 2: Ax B $\neq$ B xA

Question Set 1

3. Multiply the following matrices:

$$\begin{bmatrix} 2 & 3 & 4 \\ 0 & 10 & 3 \\ 1 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 5 \\ 5 & 6 & 9 \\ 1 & 2 & 0 \end{bmatrix}$$

4. $$\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 10 \end{bmatrix}$$

5. $$\begin{bmatrix} 3 & 4 & 2 \\ 2 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 3 & 1 \\ 4 & 2 & 2 \end{bmatrix}$$

Is it possible to compute No.5?! No! Why?

Transpose of a Matrix

- Matrix formed by interchanging rows and columns of A is called A transpose (A')

If A=$\begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$ then $A'=\begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$

Q. Verify (A B)' = B' xA'

I. If A=$\begin{bmatrix} 2 & 4 & 0 \\ 3 & 9 & 6 \end{bmatrix}$ and B=$\begin{bmatrix} 1 & 4 \\ 2 & 8 \\ 1 & 3 \end{bmatrix}$

Q. Verify (A + B)'= A' + B'

If A=$\begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$ and B=$\begin{bmatrix} 3 & 6 & 5 \\ -1 & 5 & 9 \end{bmatrix}$

Symmetric matrix: A square matrix A is called symmetric if $A^T = A$

Remark: In a symmetric matrix, the entries opposite to diagonal entries are same. $a_{ij} = a_{ji}$

EX : i} $\begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}$ ii} $\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 5 \\ 3 & 5 & 4 \end{bmatrix}$

Skew symmetric matrix : A square matrix A is called skew symmetric if A^T=-A or $a_{ij} = -a_{ji}$ .In skew symmetric matrix diagonal elements are "0" and entries opposite to main diagonal are same but opposite sign.

EX: $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ ii} $\begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & -5 \\ -3 & 5 & 0 \end{bmatrix}$

Question Set 1

6. Find the transpose of the following matrices and verify that (A+B)' = A' +B'

$A = \begin{bmatrix} 1 & 2 & 9 \\ 4 & 3 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 8 & 6 \end{bmatrix}$

Hint: Find A+B, (A+B)',A' and B' and verify

7. If D is a matrix where first row = number of table fans and second row = number of ceiling fans factories A and B make in one day. If a week has 5 working days compute 5A. What does 5A represent?

$D = \begin{bmatrix} 10 & 20 \\ 30 & 40 \end{bmatrix}$

Question Set 1

9. Two shops have the stock of large, medium and small sizes of a toothpaste. The number of each size stocked is given by the matrix A where

$$A = \begin{array}{c} \begin{array}{ccc} \text{Large} & \text{Medium} & \text{Small} \end{array} \\ \begin{bmatrix} 150 & 240 & 120 \\ 90 & 300 & 210 \end{bmatrix} \end{array} \begin{array}{l} \text{shop no. 1} \\ \text{shop no 2} \end{array}$$

The cost matrix B of the different size of the toothpaste is given by

$$B = \begin{array}{c} \text{Cost} \\ \begin{bmatrix} 14 \\ 10 \\ 6 \end{bmatrix} \end{array}$$

Find the investment in toothpaste by each shop

Answer: $\begin{bmatrix} 3820 \\ \end{bmatrix}$ **-- Investment by shop no 1 5520**

-- Investment by shop no 2

Question Set 1

8. For the matrix

$$A = \begin{bmatrix} 4 & 5 & 6 \\ 2 & 1 & 3 \\ -5 & 2 & 2 \end{bmatrix} \quad \text{and } B = \begin{bmatrix} 7 & 9 \\ 10 & 2 \end{bmatrix}$$

Multiply by the Matrix I =

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

What is A.I and I.B ?

Identity Matrix

- If you were to multiply 'a' by '1', you would get 'a' .
  Eg: 2 x 1 =2x1 =2

- The 'identity' matrix (i) is the equivalent of '1' in basic math
  If A is a matrix and I is an identity Matrix,

- Then A x I =A and I xA =A. Identity Matrices

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Then A x I =A and I xA =A

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a+0 & 0+b \\ c+0 & 0+d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

To find inverse by using elementary Row transformations.

- **Step 1:** Write **A = IA**
- **Step 2:** Apply various row operations on left hand side and apply same operations to **I** on right side but not to A on right side.
- **Step3:** From step 2 we get a new matrix equation **I = BA** . Hence **B =** $\mathbf{A^{-1}}$.

To find inverse by using elementary Column transformations.

- **Step 1:** Write **A = AI**
- **Step 2:** Apply various Column operations on left hand side and apply same operations to **I** on right side but not to A on right side.
- **Step3:** From step 2 we get a new matrix equation **I = AB** . Hence **B** = A^{-1} .

INVERSE OF ORDER 2 MATRIX

Exmples:

Using elementary row transformation find the inverse of $\begin{bmatrix} 6 & 5 \\ 5 & 4 \end{bmatrix}$.

Ans : Given $A = \begin{bmatrix} 6 & 5 \\ 5 & 4 \end{bmatrix}$

Consider $A = IA$

$$\Rightarrow \begin{bmatrix} 6 & 5 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 - R_2$

$$\begin{bmatrix} 1 & 1 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 - 5R_1$

$$\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -5 & 6 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 + R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -4 & 5 \\ -5 & 6 \end{bmatrix} A$$

Applying $R_2 \rightarrow (-1)R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -4 & 5 \\ 5 & -6 \end{bmatrix} A$$

Hence $A^{-1} = \begin{bmatrix} -4 & 5 \\ 5 & -6 \end{bmatrix}$

INVERSE OF ORDER 3 MATRIX

1. Using elementary row transformation find the inverse of $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

Ans :Given $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

Consider $A = IA$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow 3R_1$

$$\begin{bmatrix} 6 & 0 & -3 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 - R_2$

$$\begin{bmatrix} 1 & -1 & -3 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 + R_3$

$$\begin{bmatrix}1 & 0 & 0\\5 & 1 & 0\\0 & 1 & 3\end{bmatrix} = \begin{bmatrix}3 & -1 & 1\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}A$$

Applying $R_2 \rightarrow R_2 - 5R_1$

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 1 & 3\end{bmatrix} = \begin{bmatrix}3 & -1 & 1\\-15 & 6 & -5\\0 & 0 & 1\end{bmatrix}A$$

Applying $R_3 \rightarrow R_3 - R_2$

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 3\end{bmatrix} = \begin{bmatrix}3 & -1 & 1\\-15 & 6 & -5\\15 & -6 & 6\end{bmatrix}A$$

Applying $R_3 \rightarrow \frac{1}{3}R_3$

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} = \begin{bmatrix}3 & -1 & 1\\-15 & 6 & -5\\5 & -2 & 2\end{bmatrix}A$$. Hence $A^{-1} = \begin{bmatrix}3 & -1 & 1\\-15 & 6 & -5\\5 & -2 & 2\end{bmatrix}$

Inverse of a Matrix

In basic math: $2 \div 2 = 1$ and $1/2 \times 2 = 1$.

Dividing 2 by two is the same as multiplying 2 by 1/2 . The net result is 1.

A similar concept is the 'inverse' of a matrix. If A is a matrix, then A^{-1} is the inverse such that $A \times A^{-1} = I$ (identity matrix)

If A has an inverse(A^{-1}) then A is said to be 'invertible'

$$\mathbf{A.A^{-1} = A^{-1}.A = I}$$

The determinant is a scalar value that can be computed from the elements of a square matrix

$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then determinant of A or detA or $|A|$

$|A|=a_{11}(a_{22}a_{33}-a_{32}a_{23})$-$a_{12}(a_{21}a_{33}-a_{31}a_{23})+a_{13}(a_{21}a_{32}-a_{31}a_{22})\neq 0$ then inverse of matrix exist i.e A^{-1}

$A=\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 5 \\ 3 & 5 & 4 \end{bmatrix}=1(-1.4-5.5)$-$2(2.4-3.5)+3(2.5-3.-1)=11\neq 0$

here inverse of A exist. So $A^{-1}=\frac{adjA}{|A|}$, $|A| \neq 0$

$\text{adjA}=\begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}=\begin{bmatrix} -29 & 7 & 13 \\ 7 & -5 & 1 \\ 13 & 5 & -5 \end{bmatrix}$

A_{11} = -29 A_{12} =7 A_{13} = 13 A_{21}= 7 $A_{22}=-5$ $A_{23}=5$ $A_{31}=13$ $A_{32}=1$ $A_{33}=-5$

$$A^{-1}=\frac{adjA}{|A|}=\frac{1}{11}\begin{bmatrix} -29 & 7 & 13 \\ 7 & -5 & 5 \\ 13 & 1 & -5 \end{bmatrix}=\begin{bmatrix} \frac{-29}{11} & \frac{7}{11} & \frac{13}{11} \\ \frac{7}{11} & \frac{-5}{11} & \frac{1}{11} \\ \frac{13}{11} & \frac{5}{11} & \frac{-5}{11} \end{bmatrix}$$

If the simultaneous equation are of the form

$a_{11}x+a_{12}$y$+a_{13}$z$=b_1$ it can also be represented in matrix form AX=B

$a_{21}x+a_{22}$y$+a_{23}$z$=b_2$ where $A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ $X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $B=\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

$a_{31}x+a_{32}$y$+a_{33}$z$=b_3$ so $X=A^{-1}$ B $=\frac{adjA}{|A|}$ B

EX : Solve by matrix method x+y+z=6 ,2x-y+5z=6 ,3x+5y+4z=12

Ans Here $A=\begin{bmatrix}1 & 2 & 3\\ 2 & -1 & 5\\ 3 & 5 & 4\end{bmatrix}$ $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$ $B=\begin{bmatrix}6\\ 6\\ 12\end{bmatrix}$

$|A|=\begin{vmatrix}1 & 2 & 3\\ 2 & -1 & 5\\ 3 & 5 & 4\end{vmatrix}=1(-1.4-5.5)-2(2.4-3.5)+3(2.5-3.-1)=11\neq 0$

So inverse exist , Previous example it is found that

$$A^{-1}=\frac{adjA}{|A|}=\frac{1}{11}\begin{bmatrix}-29 & 7 & 13\\ 7 & -5 & 5\\ 13 & 1 & -5\end{bmatrix}=\begin{bmatrix}\frac{-29}{11} & \frac{7}{11} & \frac{13}{11}\\ \frac{7}{11} & \frac{-5}{11} & \frac{1}{11}\\ \frac{13}{11} & \frac{5}{11} & \frac{-5}{11}\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\\ z\end{bmatrix}=X=A^{-1}B=\begin{bmatrix}\frac{-29}{11} & \frac{7}{11} & \frac{13}{11}\\ \frac{7}{11} & \frac{-5}{11} & \frac{1}{11}\\ \frac{13}{11} & \frac{5}{11} & \frac{-5}{11}\end{bmatrix}\begin{bmatrix}6\\ 6\\ 12\end{bmatrix}=\begin{bmatrix}1\\ 1\\ 1\end{bmatrix}$$

So x=1,y=1 and z=1

Singular matrix : A square matrix is said to be singular if det(A)=0

Ex : Let $A=\begin{bmatrix}2 & 4\\ 1 & 2\end{bmatrix}$ here det(A)=4-4=0 , so given matrix is singular.

Non singular Matrix : A square matrix is said to be non singular if det(A) $\neq$0,

Note : Inverse of a square matrix exist in non singular matrix.

KEY POINTS

- A matrix is an ordered rectangular array of numbers or functions.
- A matrix having m rows and n columns is called a matrix of order mxn
- A is a diagonal matrix if its non diagonal elements are zero.
- A is a identity matrix if diagonal elements are 1 and non diagonal elements are 0
- A is zero matrix if all elements are zero.
- Matrix addition is commutative ,associative over same order. A+B= B+A, (A+B)+C=A+(B+C)

KEY POINTS

- k (A+B)= kA+kB , k is constant A,B are of same order
- If order of first matrix A is m and that of B is n, then A.B is possible of order m
- Matrix multiplication is not commutative.
- A matrix is symmetric if $a_{ij} = a_{ji}$
- If a square matrix is invertible if detA≠0, or singular matrix
- if detA =0,inverse of a square matrix doesn't exist.or non singular matrix

$$(kA)^T = kA^T$$
$$(A \pm B)^T = A^T \pm B^T$$
$$(A^{-1})^T = (A^T)^{-1}$$
$$(A^T)^T = A$$

CONCEPT MAPPING

Order of a Matrix
A matrix having m rows and n columns is called a matrix of order $m \times n$.

Transpose
Transpose is obtained by interchanging rows and columns. If $A = [a_{ij}]_{m\times n}$ then A' or $A^T = [a_{ji}]_{n\times m}$

Properties

$(A')' = A$

$(AB)' = B'A'$

$(A \pm B)' = A' \pm B'$

$|A'| = |A|$

Types of Matrix

Column Matrix : $A = [a_{ij}]_{m \times 1}$

Row Matrix : $A = [a_{ij}]_{1 \times n}$

Square Matrix : $A = [a_{ij}]_{m \times m}$

Diagonal Matix : $A = [a_{ij}]_{m \times m}$ where $a_{ij} = 0 \ \forall \ i \neq j$

Scalar Matrix : $A = [a_{ij}]_{n \times n}$ where $a_{ij} = \begin{cases} 0, \text{ if } i \neq j \\ k, \text{ if } i = j \end{cases}$ for some constant k

Zero Matrix : $A = [a_{ij}]$, where $a_{ij} = \{0 \ \forall \ i = j$ and $i \neq j$

Identity Matrix : $A = [a_{ij}]$ where $a_{ij} = \begin{cases} 1 \text{ if } i = j \\ \ldots \end{cases}$

Special Matrices

Nilpotent Matrix : $A^k = 0$ and $A^{k-1} \neq 0, k \in z^+$ $\Rightarrow |A| = 0$, order = Least value of k

Involutory Matrix : $A^2 = I \Rightarrow |A| = \pm 1$

Orthogonal Matrix : $AA^T = A^T A = I$ $\Rightarrow |A| = \pm 1$

Periodic Matrix : $A^k = A$ $\Rightarrow |A| = 0, 1$, order $= k - 1$

Idempotent Matrix : $A^2 = A \Rightarrow |A| = 0, 1$

Unitary Matrix : $AA^\theta = A^\theta A = I$

Symmetric Matrix : $A' = A$

Skew Symmetric Matrix : $A' = -A$

Inverse

If A and B are two square matric[es] such that $AB = BA = I$, then B is th[e] inverse matrix of A and is denote[d] by A^{-1} and A is the inverse of B.

DAV PUBLIC SCHOOL POKHARIPUT,
SUBJECT- MATHEMATICS,CLASS- XII
CHAPTER-3 (MATRIX)
WORKSHEET(BASIC)

Group –A **[Objective type]**

1. Find the values of x, if $\begin{bmatrix} 3x+y & -y \\ 2y-x & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -5 & 3 \end{bmatrix}$.
2. If A is symmetric and skew symmetric then A is ....
3. A square matrix A is said to be skew-symmetric if ....
4. If A is a matrix of order 3×2 , then the order of the matrix A' is ....
5. For a 3×3 matrix, A = $[a_{ij}]$ whose elements are given by $a_{ij} = \frac{|i-j|}{2}$, write the value of a_{23} .
6. Write order of product matrix $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$.
7. If a matrix has 5 elements, then write all possible orders it can have
8. If A is a square matrix such that $A^2 = A$, then write value of $7A-(A+I)^3$.
9. If A and B are square matrices such that ab=ba, find the value of $(A+B)^2$
10. If A and B are square matrices of same order, find $(A+B)(A-B)$
11. Let A and B be two matrices of order 3× 2 and 2× 4 , then write the order of AB matrix.
12. Write the order of i} $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & -1 \end{bmatrix}$ ii} $[-1]$
13. If matrix A = $[1 \quad 2 \quad 3]$, then write $A.A'$.
14. Write the number of all possible matrices of order 2×2 with each entry 1, 2
15. For a 2×2 matrix, A = $[a_{ij}]$ whose elements are given by $a_{ij} = \frac{i}{j}$,write the value of a_{12} .
16. Find the transpose of matrix $\begin{bmatrix} -1 & 9 & 5 \\ 2 & 8 & 5 \\ 3 & 5 & 7 \end{bmatrix}$
17. If 3A-B=$\begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix}$ and B $\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$ find the value of A

18. If A=$\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ then find value of AA'

19. If $A'=\begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$then find A'-B'

20. If $\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix}=\begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$, then find value of x+y.

21. If A is a matrix of order 3×4 and B is a matrix of order 4×3 , then find the order of matrix AB.

22. If $\begin{pmatrix} 2x+y & 3y \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ 6 & 4 \end{pmatrix}'$, then find the value of x .

23. If A is a matrix of order 3×2 , then the order of the matrix A' is ....

24. For what value of x, is the matrix $\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ a skew-symmetric matrix .

25. If $\begin{bmatrix} x.y & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$, then write the value of (x + y + z) .

26. Write the number of all possible matrices of order 2×2 with each entry 1, 2 or 3

27. Write order of product matrix $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$.

28. If A and B are of same order write $(AB)'$ is ....

29. Construct a matrix A is of order 2 such that a_{ij}= 2 for i≠ j and a_{ij}= 0 for i=j.

30. If A and B are of same order write $(AB)'$ is ....

WORKSHEET(STANDARD)

Group – A **[Each 2 marks]**

1. For the matrix $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$, find $A + A'$ and verify that it is a symmetric matrix.

2. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $I = = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find "k" so that $A^2 = 7A + kI$.

3. Find the value of a,b,c and d if $\begin{bmatrix} a+b+c+d \\ a+c-d \\ b-c+d \\ a+d \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 1 \\ 2 \end{bmatrix}$

4. If $X = \begin{bmatrix} 3 & 1 & -1 \\ 5 & -2 & -3 \end{bmatrix}$ $and\ Y = \begin{bmatrix} 2 & 1 & -1 \\ 7 & 2 & 4 \end{bmatrix}$ then find matrix Z, such that X+Y+Z is a zero matrix.

5. Find x from the matrix equation $\begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} x \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$

6. If $\begin{bmatrix} xy & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$ then write the value of $(x + y + z)$

7. Show that all the diagonal elements of a skew-symmetric matrix are zero

8. If $A = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$ then find value of x for which $A^2 = B$

9. If $\begin{bmatrix} xy & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$ then write the value of $(x + y + z)$

10. Show that all the diagonal elements of a skew-symmetric matrix are zero .

11. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $I = = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that $A^2 = 5A + kI$.

12. If A and B are symmetric matrices such that AB and BA are both defined, then prove that AB-BA is a skew symmetric matrix.

13. For the following matrices A and B, verify that $[AB]' = B'A'$;

$A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$, $B = [-1 \quad 2 \quad 1]$.

14. Using elementary row transformations, find the inverse of $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$.

Group – B [Each 4 marks]

15. Express the matrix A = $\begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrices and verify your result.

16. Using elementary row transformations, find the inverse of $\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$

17.If A=$\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ then prove that $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$

18.If A=$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ then verify that $A^2 - 4A - 5I = O$

19.If A=$\begin{bmatrix} 2 & 4 & 0 \\ 3 & 9 & 6 \end{bmatrix}$ and B=$\begin{bmatrix} 1 & 4 \\ 2 & 8 \\ 1 & 3 \end{bmatrix}$ then verify that $(AB)' = B'A'$

20.Solve the matrix $\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 1 \\ 4 \end{bmatrix} = O$

WORKSHEET(ADVANCED)

1. Using elementary row transformations, find the inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$.
2. Show that a matrix which is both symmetric as well as the skew symmetric matrix is a null matrix .
3. If f(x) = $\begin{bmatrix} cosx & -sinx & 0 \\ sinx & cosx & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then show that $f(x)$.f(y)=f($x+y$)
4. Use matrix multiplication to divide Rs 30000 in two parts such that the total annual interest at 9% on the first part and 11% on the second part amounts Rs3060.
5. Find the matrix A such that A$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$=$\begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$
6. Show that A=$\begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$ satisfies the equation $x^2-6x+17$=0. Also find A^{-1}
7. Let A= $\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ B=$\begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$ and C= $\begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$ Find a matrix D such that CD-AB=O
8. Find the matrix A satisfying the matrix equation $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
9. If A =$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ find A^{-1}. Hence solve the system of equations 2x-3y+5z=11 3x+2y-4z=-5 and x+y-2z=-3
10. If A= $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ then prove that A is a root of the polynomial f(x)=x^3-6x^2+7x+2
11. The cost of 4kg onion, 3kg wheat and 2kg rice is Rs60. The cost of 2kg onion, 4kg wheat and 6kg rice is Rs90. The cost of 6kg onion, 2kg wheat and 3kg rice is Rs70 .Find the cost of each item per kg by matrix method
12. If AB= BA for any two square matrices, then prove by mathematical induction that $(AB)^n$=A^nB^n

NAMASKAR